EXAMPLES IN
APPLIED MATHEMATICS

BY

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PREFACE

This collection, containing just over 400 examples divided into 20 convenient sections, is designed to provide practice and a testing-ground for students preparing for such examinations as the Qualifying Examination of the Mechanical Sciences Tripos (Cambridge), the Intermediate B.Sc. Engineering (London), and the Intermediate Arts and Science (London), and for those doing similar work.

Questions requiring almost entirely "bookwork" or its immediate applications (e.g. those necessitating nothing but substitution of numbers in a formula) have been excluded. A good variety of problems in each section, without undue repetition, has been an objective.

Many of the questions are original; others have been selected from various examination papers. For these, thanks are accorded to the Syndics of the Cambridge University Press, the Universities of London and Reading, the Oxford and Cambridge Schools Examination Board, and (for examples chosen from Board of Education examination papers) to the Controller, H.M. Stationery Office.

My thanks are due also to a number of friends, former students, who gave assistance in checking results; in particular to Messrs. P. J. Crowther, P. C. Hodge, Whitworth Scholar, D. W. Lang, B.Sc.(Eng.), Royal Scholar, and Const.-Lieut. W. H. Thomas, R.N.

Finally, I wish to express my sincere gratitude for the help given me by the publishers at all stages of the work.

F. W. K.

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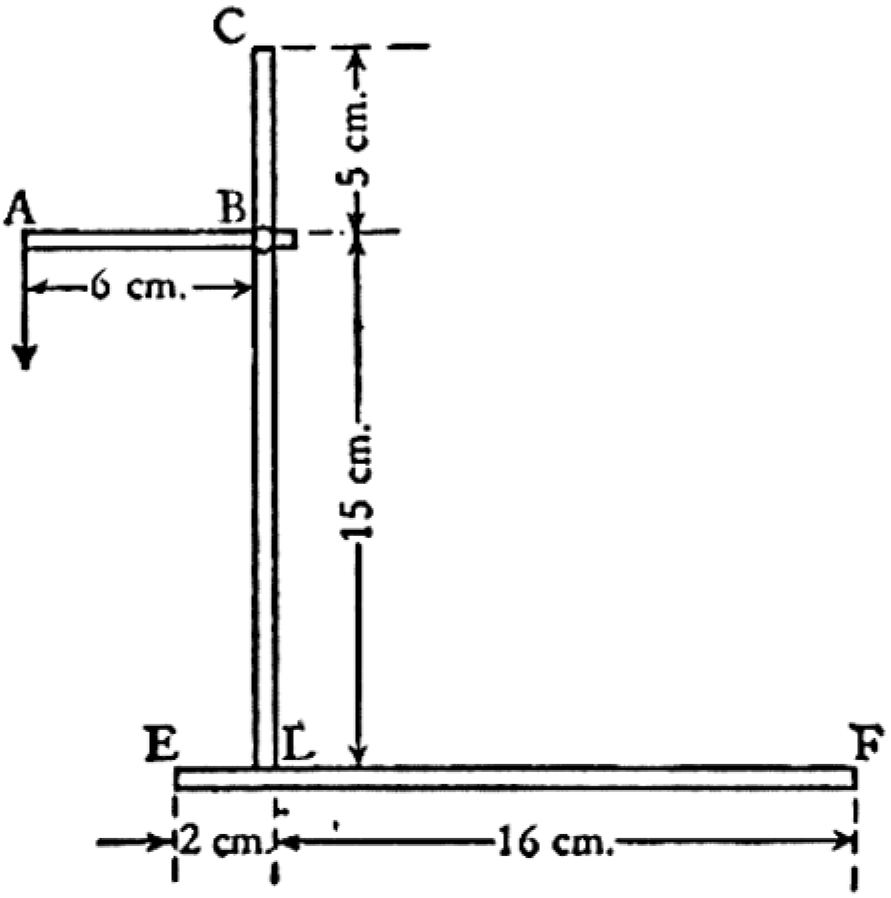
STATICS

SECTION A

1. The figure depicts a retort-stand consisting of a vertical rod CD fixed to a base EF, and an arm AB. CD has mass 40 gm., EF 100 gm., and AB 10 gm., the centre of gravity of each piece being at its mid-point. Neglecting the thickness of each piece, find what load can be suspended from A without causing the stand to tilt about E.

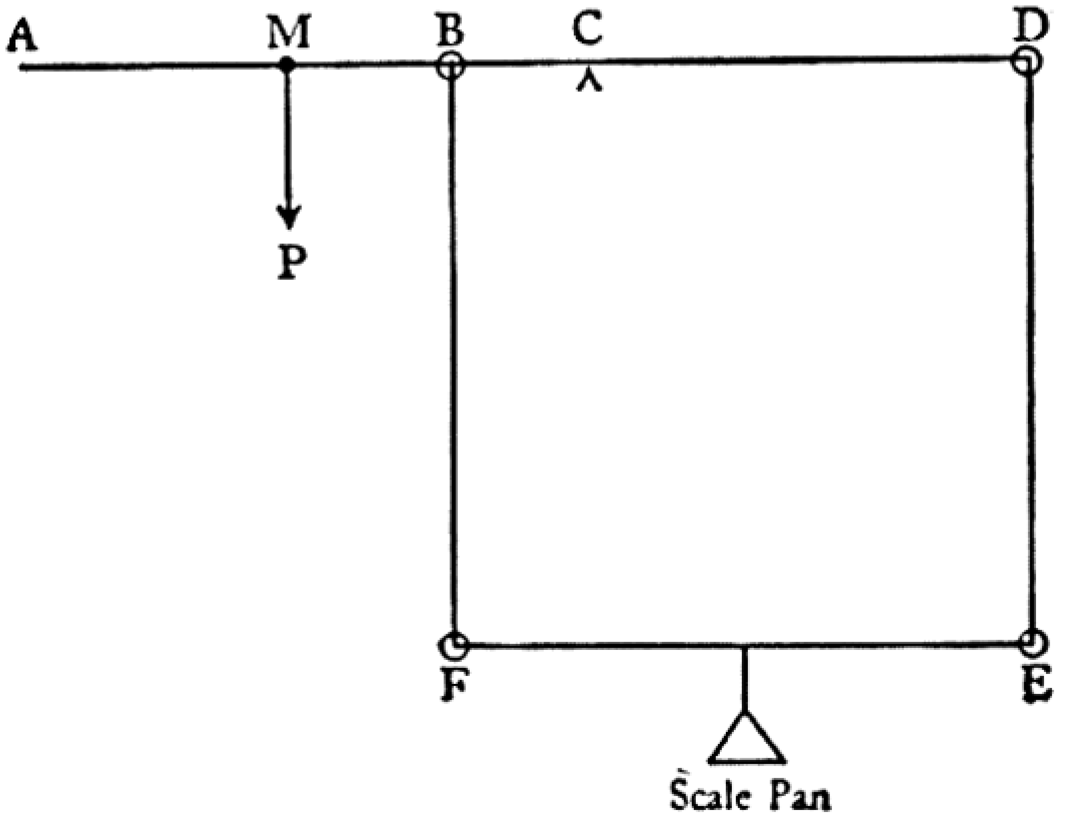


2. Two boys, weighing 50 and 65 lb.-wt., use a plank, length 8 ft., as a see-saw. Suppose that they sit so that their weights act at points 6 in. from the end of the plank. About what point should the plank be balanced? Alternatively, if the plank is balanced about its mid-point and the smaller boy remains 6 in. from one end, where should the other boy sit? Consider the cases (*a*) when the weight

of the plank is neglected; (*b*) when the plank weighs 100 lb.-wt.

3. A straight uniform rod, length 12 ft., mass 14 lb., rests horizontally on two supports, one at one end and the other 7 ft. from that end. Two masses, each of 7 lb., are placed at the points of trisection of the rod. Find the reactions at the supports. By how much are these reactions increased if an additional mass of 7 lb. is placed at the centre of the rod?

4. A weighing machine is constructed as shown in the figure. A stiff rod ABCD is pivoted at C, BC being less than CD. Equal rods BF, DE are suspended from B and D; a rod FE, equal in length to BD, joins their ends, and a scale pan is attached to the mid-point of FE. The rods are freely jointed at B, D, E, F. The counterpoise is a weight P, which is in a position M on AB when the machine is in equilibrium, unloaded. Explain how to graduate the machine.



5. A uniform metre stick, mass 50 gm., is suspended horizontally by two vertical strings attached to the 10 cm. and 90 cm. marks. If each string breaks under a tension of 45 gm.-wt., between what marks on the scale may a mass of 30 gm. be attached without a string breaking?

6. A uniform rod AB, which is 6 ft. long and weighs 8 lb.-wt., is suspended horizontally by means of two vertical strings attached to the rod at B and C, where AC $= x$ ft. A mass of 10 lb. is attached to the rod at a distance of $2\frac{1}{2}$ ft. from A. Find the tension in each string.

If the breaking tension of each string is 12 lb.-wt., find the greatest possible length of AC.

7. AB is a uniform rod 6 ft. long, weighing 32 lb.-wt.; BC is a uniform rod 4 ft. long, weighing 20 lb.-wt. The rods are freely jointed together at B and rest in equilibrium in a horizontal straight line. If the support on which BC rests is $1\frac{1}{2}$ ft. from C, find the position of the support on which AB rests.

8. A uniform bar ACDB, length 80 cm., mass 120 gm., rests on two supports C and D, 30 cm. apart, and a load of 40 gm.-wt. is hung from A. The bar is pushed across the supports in the direction D to C. Find its position (i) when the supports carry equal loads, (ii) when the bar is on the point of toppling over.

9. Masses of 3 lb. and 7 lb. are hung at A and B, the ends of a heavy uniform rod, length 3 ft., mass 6 lb. The rod is supported in a horizontal position by two pegs, one at a distance of $2\frac{1}{2}$ in. from B. Find the possible positions of the second peg if the pressure on one of the pegs is double that on the other.

10. AB is a uniform straight rod, length 20 ft., mass 60 lb., which is supported in a horizontal position by vertical forces at A and B. Masses of 10 lb. and 20 lb. are suspended from points at distances of 3 ft. and 8 ft. respectively from A. Find the load which must be attached to a point 4 ft. from B in order that the reactions at A and B may be in the ratio of 2 to 3.

11. A uniform plank ABCD, length 12 ft., mass 24 lb., rests horizontally in equilibrium by passing over a support at B and under a peg at C, a load of 16 lb.-wt., being suspended from A. If BC $= 6$ ft. and the magnitude of the

reaction at B is six times the magnitude of the reaction at C, find the length of AB.

12. Three straight uniform rods, of the same material, AB, BC, CD, have lengths 15, 3, 6 ft. respectively. They are smoothly hinged at B and C, and supported at A, D and a point P in BC by three vertical strings. The rods being in one horizontal straight line, find the distance BP.

13. A uniform beam, weight W, length $3b + c$, rests horizontally on two props distant c apart so that a length b projects beyond one prop. A man of weight nW stands on the beam at a distance of x from the end projecting the length b. Find the reactions at the props. Find also the minimum value of c (expressed in terms of n and b) in order that the man can walk from one end of the beam to the other without its tilting.

14. A uniform plank AB, 6 ft. long and of given weight, is supported horizontally by two pegs at C and D respectively, where AC = BD = 8 in. Another exactly equal plank A′B′ is placed over the first plank so that A′ projects 11 in. beyond A. Find the pressures R_1 and R_2 at C and D.

A vertical force P is applied at A′, but so that equilibrium is still maintained; if R_3, R_4 are the new pressures at C and D, show that

$$(R_3 - R_1) / (R_2 - R_4) = \tfrac{75}{19}.$$

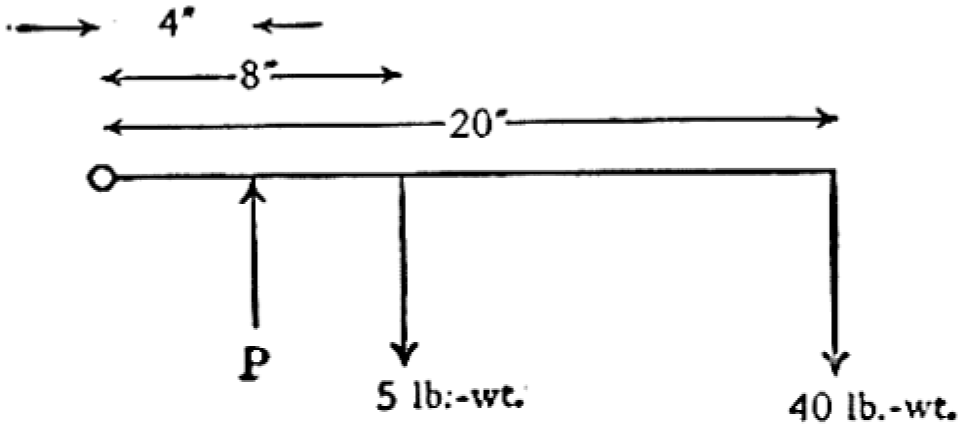
15. A uniform rod AB, weight W, rests horizontally on props at M, N, where AM $= \frac{1}{3}$ AB, AN $= \frac{3}{5}$ AB. The rod remains at rest when weights P and Q hang from A and B. Prove that

$$(6P + W) > 4Q > (2P - W).$$

If $Q = P$, find the value of the additional vertical force applied at B which will just disturb equilibrium. In which direction (i.e. upwards or downwards) will this force apply?

SECTION B

1. The weight of a lever safety valve, mass 5 lb., can be considered as acting at a point 8 in. from the fulcrum (see figure). A mass of 40 lb. is suspended from the end of the lever, 20 in. from the fulcrum. The valve, of $1\frac{1}{4}$ in. radius, has its centre 4 in. from the fulcrum. What pressure of steam will cause the lever to move?



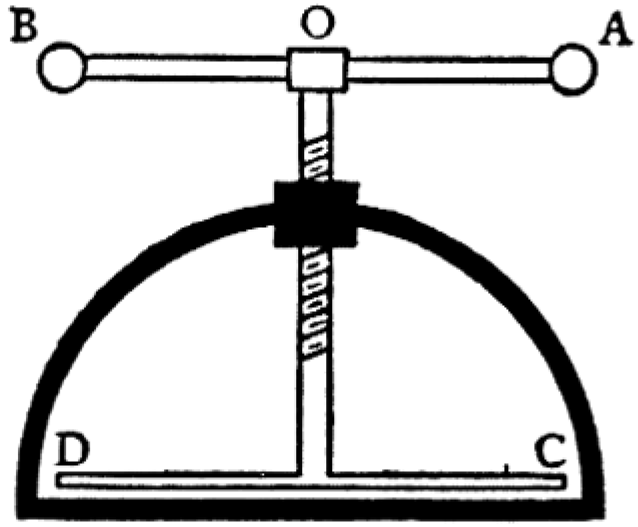
2. A pulley system consists of two blocks each containing two pulleys. Show how they can be arranged to give a velocity ratio of 4 : 1 or 5 : 1. If the efficiency of the first arrangement is 70 per cent for a load of 150 lb.-wt., what is the mechanical advantage?

3. Four equal pulleys, arranged so that the strings are connected to a bar 6 in. long at points $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, and $4\frac{1}{2}$ in. from one end, are used to move a mass which is suspended from the bar. If the bar is to be always horizontal, from where must the mass be hung?

4. The radii of a wheel and axle are respectively 3 ft., 1 ft., and the efficiency is 60 per cent. Find the effort necessary to raise a mass of 200 lb. and the greatest load which can be raised with an effort of 60 lb.-wt.

5. AB is a rod, loaded at A so that the distance of the centre of gravity, G, from A is one-tenth of AB; the rod can turn freely round an axis at C, which is half-way between A and G. The mass of the rod is 4 lb., a mass of 10 lb. is hung from A and a mass of 1 lb. is placed at a point P so as to restore the equilibrium; find AP as a fraction of AB.

6. An effort of 4 lb.-wt. acting on an arm 1 ft. long supports a mass of 1 cwt. on the top of a smooth vertical screw, diameter 2 in. Find the pitch of the screw.



7. In the screw-press in the figure the pitch of the screw is $\frac{1}{3}$ in. and the length of each arm OA and OB is 2 ft. Prove that the velocity ratio of the press is 144π. A man pulls the handle A towards him with one hand and pushes B away from him with the other, exerting the same horizontal force perpendicular to AB with each hand. What is the force exerted by one hand, if the mechanical efficiency is 70 per cent and the thrust produced by the press is equal to the weight of 5 cwt.?

8. The plunger of a Bramah press, diameter 1 in., is operated by a lever whose velocity ratio is 12 : 1. The diameter of the ram is 10 in., the efficiency of the press 85 per cent. Find the force exerted by the ram when a force of 35 lb.-wt. is applied to the handle of the lever.

9. The diameters of the pistons in a hydraulic press are 2 in. and 14 in. Find the effort that, applied by a lever system having mechanical advantage 6, develops a total thrust of 2 tons-wt.

10. The diameter of the chain drum of a crane is 9 in.; it is keyed to a large cogwheel containing 114 teeth, which is geared to a small cogwheel of 11 teeth on whose shaft is a wheel of 48 teeth turned by a wheel of 13 teeth which is itself turned by a crank handle of 18 in. crank radius. Find the velocity ratio of the machine.

11. A balance having light arms of unequal length, and scale pans of unequal weight, does not balance when unloaded. A body known to weigh a lb.-wt. appears to weigh x lb.-wt.; another body known to weigh b lb.-wt. appears

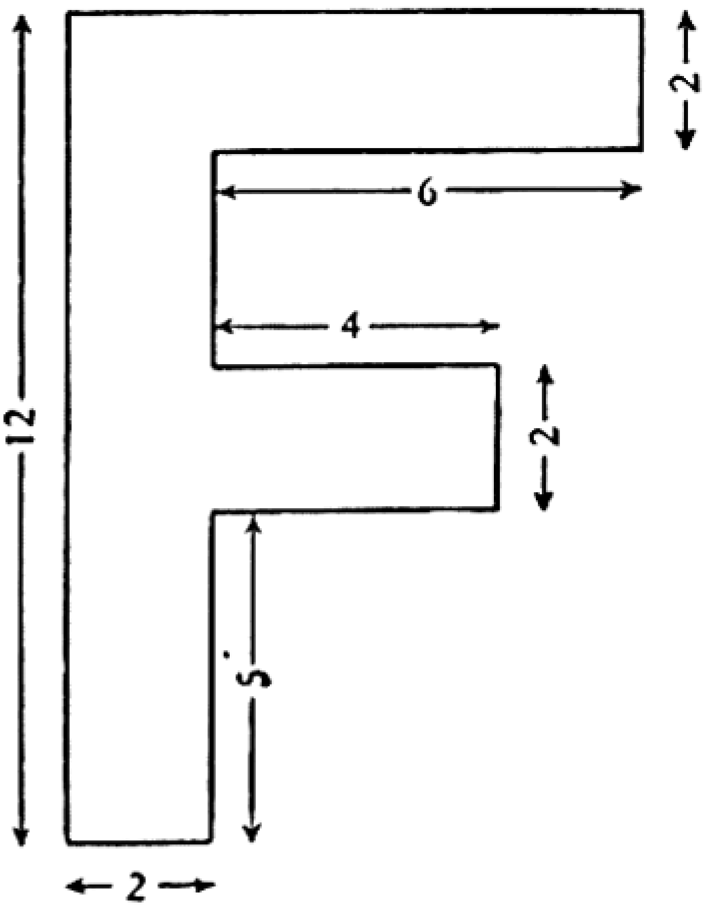
to weigh y lb.-wt. Show that the true weight of a body which appears to weigh z lb.-wt. is

$$[bx - ay + (a - b)z] \,/\, (x - y) \text{ lb.-wt.}$$

12. A faulty balance has arms of unequal length and pans of unequal weight, but is in equilibrium when the pans are not loaded. M, placed in one pan, is balanced by m_1 in the other pan; and when M is in the second pan, it is balanced by m_2 in the first pan. Show that $M^2 = m_1 . m_2$. If the pans are interchanged, m_3 and m_4 are required for balancing. Find the ratio of the lengths of the arms.

13. E, F are two fixed pulleys, A, B, C, D are movable pulleys. A fine string with one end fixed passes under A, over E, under B and over F, and carries a mass P at its other end. Another string has its ends attached to A and E and carries C in its loop; a third string has its ends attached to C and B and carries D in its loop. All the hanging parts of the strings are vertical. A has mass 8 lb., B 4 lb., C 2 lb. Prove that, if the system is in equilibrium, D and P are of determinate masses, and find their values.

14. The dimensions, in inches, of a plate in the shape of

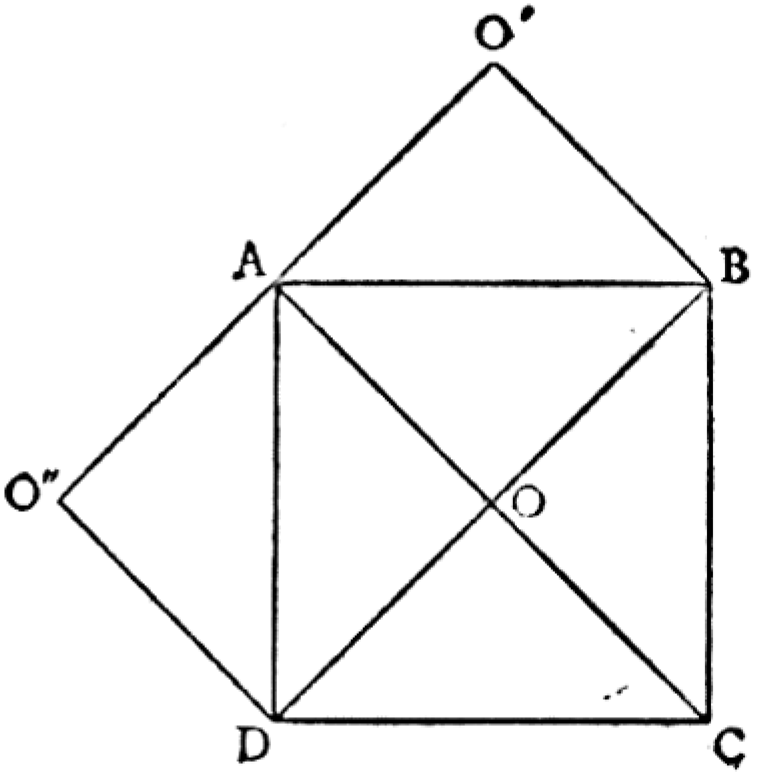


a letter F are shown in the figure. Find the position of the centre of gravity of the plate.

15. ABC is a triangular lamina, right-angled at C, with centre of gravity at G. Prove that $AG^2 + BG^2 = 5\,CG^2$.

16. A uniform rectangular door, height 7 ft., breadth 4 ft. 6 in., weighs 50 lb.-wt. A circular piece of radius 6 in., with centre 3 ft. from the top edge of the door, is removed and a glass window weighing 6 lb.-wt. inserted. Find the height of the centre of gravity of the door above the bottom horizontal edge.

17. The top of a square card table consists of two equal boards, the upper one being divided into four along the diagonals AC, BD, and being hinged to the lower one along AB, BC, CD, DA. The flaps AOB, AOD are opened out as in the figure, and a candle-stick of mass equal to that of a flap is placed at the corner O′ of the flap AO′B. Find the distances from AB and AD of the centre of gravity of the table thus loaded, if AB equals 1 foot.



18. A solid right circular cylinder and a solid right circular cone of the same material and equal base radius are fastened symmetrically together base to base. Their joint centre of gravity is in the common surface. Find the ratio of the heights of cylinder and cone.

19. A solid hemisphere and a solid right circular cone of base radius equal to the radius r of the hemisphere are

fastened together base to base. They are of the same material and their joint centre of gravity is in the common surface. Find the height of the cone.

20. A cylindrical pipe, inner and outer radii a and b, length r, has at one end a flange of outer radius c and thickness t. Find the distance of the centre of gravity of the pipe from this end, taking the material as of uniform density. Obtain a numerical result for the case when a, b, r, c, t have values 3, 3·5, 24, 6·5, 0·1 cm. respectively.

21. From the corner A of a square uniform card ABCD, edge 6 in., a square portion, edge $1\frac{1}{2}$ in., is cut out, covered uniformly on one side with half its own weight of paste, and pasted to the card in the corner B. Find the distances of the centre of mass of the body so formed from the lines AB and AD.

22. A chessboard, 8 in. square, is divided into 64 squares of side 1 in. The eleven squares whose centres are nearest one of the corners are cut away. Find the distance of the centre of gravity of the remainder from the corner opposite to that removed.

23. A square plate OXAY, side 1 ft., has two circular holes punched in it. One hole has radius 1 in. and centre at the point (4, 5), where the co-ordinates express distances in inches from OY and OX respectively. The second hole has radius $\frac{1}{2}$ in. and centre at the point (8, 4). Find the co-ordinates of the centre of gravity of the remainder.

24. In a triangle ABC the centre of BC is D. A straight line is drawn, through the centre of mass of the triangle, parallel to BC and cutting AB, AC in E, F. Show that the centre of mass of BEFC lies in AD and find its distance from D in terms of the length of AD.

25. An equilateral triangular lamina ABC has its plane vertical and AC horizontal. P and Q are points in AB and AC such that $\frac{AP}{PB} = \frac{AQ}{QC} = \frac{x}{1}$, and the centre of mass of the

trapezium BPQC is vertically above Q. Show that the value of x is $\frac{1+\sqrt{5}}{2}$.

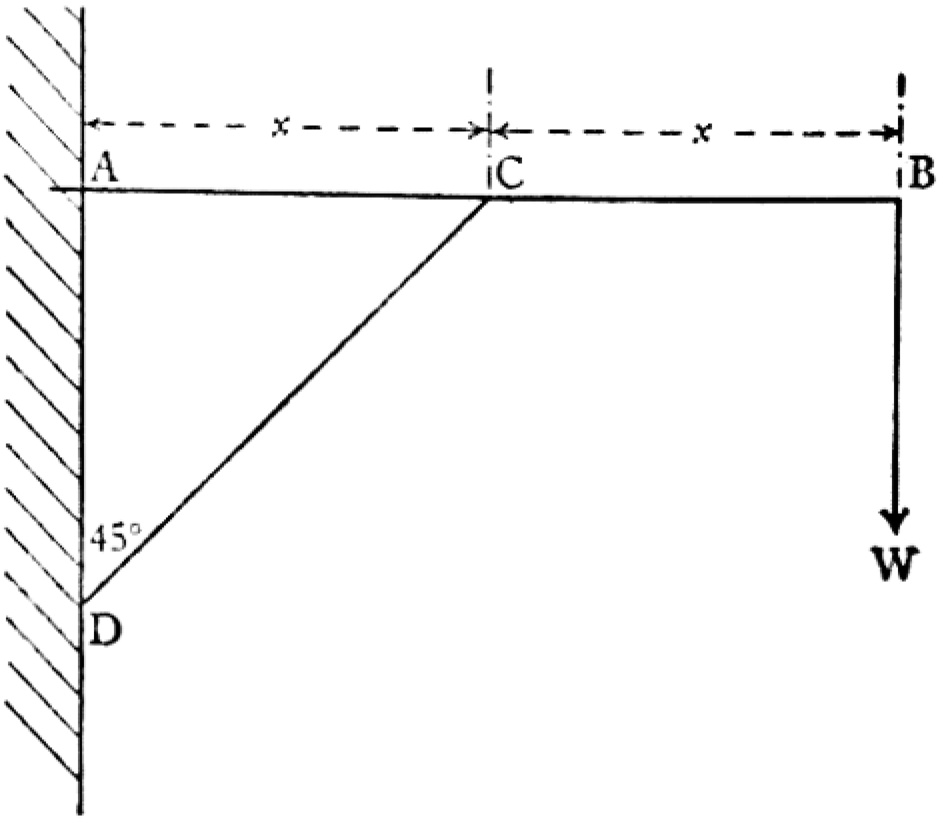
SECTION C

1. O is the centre of a regular hexagon ABCDEF. Forces of 1, 2, 3, 4, 5 and 6 lb.-wt. act along OA, OB, OC, OD, OE and OF. Find their resultant.

2. From the top of a telegraph pole six wires radiate in directions East, North-East, North, 30° West of North, 30° South of West and 10° West of South. The horizontal forces in the wires, pulling away from the pole, are in the ratio $2:a:1:3:b:4$ respectively. The net horizontal pull on the pole being zero, find the values of a and b.

3. A mass of 1 lb. is suspended by a string, and a force acts on the mass so that the string is inclined at 30° to the vertical. Find the least value of the force, and the corresponding tension in the string.

4. AB is a rod projecting horizontally from A, a point in a wall (see figure). D is a point in the wall below A, and C is the mid-point of AB; CD is a strut with smooth attachments at C and D. Find, in terms of the load W, the



thrust of the strut and the force tending to pull AB away from the wall.

5. A uniform rod AB, mass 14 lb., length 20 in., is freely hinged at A and supported by a string fastened to B and passing over a pulley P fixed at a height of 24 in. vertically above A, a mass of 8 lb. being attached to the free end of the string. Show that the angle PAB can have either of two values, and find them.

6. A rod AB, whose centre of gravity is at G, is free to turn about a horizontal axis through A. It is supported at an angle of 60° to the vertical by a string attached to a point Q of its length and to a point P vertically above A. If the point Q is chosen so that the reaction at A may be a minimum and $AG = \frac{1}{2}AP = 6$ in., find the length AQ.

7. A cylindrical roller, diameter 3 ft., mass 200 lb., with a balanced handle, is to be pulled up over a smooth kerb 6 in. high. Find the best direction of pull, and the least force necessary. At what angle with the horizontal would the pull be if its magnitude were 200 lb.-wt.?

8. A smooth uniform ladder rests with its ends against a vertical wall and a horizontal plane, and is held by a rope attached to a rung of the ladder one-fourth of the way up and to a point at the base of the wall vertically below the top of the ladder. Find the ratio of the reactions between the ladder and the ground and wall in terms of the distances x and y of the ends of the ladder from the base of the wall. Show that the reactions are equal if $x = 10$ ft. and $y = 6$ ft.

9. Two uniform rods AB, AC, each of length $2a$, mass W, smoothly jointed at A, rest symmetrically in equilibrium over a smooth cylinder, radius $\frac{3a}{2}$, having its axis horizontal. B and C are connected by a light inextensible string, length $2\sqrt{2}a$. Find the tension of the string and show that the reaction at the hinge is zero.

10. Two particles, masses 2W, 3W, fastened to the ends

of a light inextensible string which passes over two smooth pegs on the same level distant a apart, are kept in equilibrium by a third particle, mass W′, fastened to the part of the string between the pegs. If the angle between the oblique portions of the string is 120°, find W′ and its depth below the level of the pegs.

11. Two equal uniform rods, each of length a, are fixed rigidly at right-angles and are in equilibrium in a vertical plane with a free end of one of them in contact with a smooth vertical wall and the other rod resting on a smooth peg. The angle of inclination to the vertical of the former being 30°, find the distance of the peg from the wall.

12. The ends of a string, length 17 ft., are attached to two points A, B in the same horizontal line 13 ft. apart. A ring, mass W, slides on the string and a force P applied to the ring makes an angle θ with the upward vertical. The ring comes to rest so that the two parts of the string are perpendicular. Show that

$$P = \frac{7W}{7 \cos\theta + 17 \sin\theta}.$$

13. Two uniform ladders are between two smooth parallel vertical walls, in a vertical plane perpendicular to the walls, with their lower extremities in contact on a smooth horizontal plane and their upper extremities against the walls. If the lengths of the ladders are 18 and 32 ft., and they are at right angles to one another when they are in equilibrium (the weight of both ladders per foot being the same), find the distance between the walls.

14. Two uniform rods, AB, BC, lengths 3 and 4 ft., masses 7·5 and 10 lb., are smoothly jointed together at B. They rest in a vertical plane with A and C on a smooth horizontal plane, being maintained in equilibrium by a light inelastic string of length 5 ft., joining A and C. Find the tension in this string.

15. A homogeneous sphere, mass W, radius r, rests on a smooth inclined plane and is kept in position by a smooth

horizontal rail placed symmetrically at a perpendicular distance l from the plane. Find the pressure on the rail. If W and r are constant, find the value of l for which this pressure is a minimum.

16. Two heavy uniform rods, AB, AC, each of mass W, are smoothly hinged together at A and joined by a light string BC of length equal to that of one of the rods. If the system is suspended from B, find the inclination of AC to the vertical and the tension in the string.

17. A heavy uniform rod, length 4 ft., is hung from a fixed point by two light inextensible strings, lengths 7 ft. and 5 ft. If it is in equilibrium, show that the cosine of the angle between rod and vertical is $\sqrt{3/11}$.

18. A string, length $7x$, passes over a smooth peg and has its ends fastened to the ends of a rod, length $5x$, whose centre of gravity divides the rod in the ratio 3 : 4. Show that there is a position of equilibrium in which the angle between the two portions of the string is a right angle. What is then the inclination of the rod to the vertical?

19. A uniform rod AB, length r, mass W, rests between two smooth inclined planes which are inclined to the horizontal at 30° and 60° respectively, the end A being on the former plane. The rod is horizontal when a mass P is suspended from a point C in it. Prove that the distance $\mathrm{AC} = \dfrac{(\mathrm{P} - \mathrm{W})r}{4\mathrm{P}}$.

20. A uniform square lamina ABCD, mass W, area 25 sq. ft., can turn freely in a vertical plane about the point A, which is fixed. A is 4 ft. in front of a smooth vertical wall perpendicular to the plane of the lamina. The lamina rests with B against the wall. Find the reactions at A and B; and the least weight which, suspended from D, will make the lamina turn away from the wall.

21. One end of a cord is fastened to a fixed point A, while the other end is carried over a fixed pulley at B, and fastened to a mass W; what load must be attached to the

point, C, of the cord midway between A and B so that there may be equilibrium with the part AC horizontal? Show also that the tension of AC will be

$$\frac{2\cos a - 1}{\sqrt{(5 - 4\cos a)}} W$$

when AB is inclined at angle a to the horizontal.

22. A cord, 50 ft. long, whose mass is negligible, is fixed to two supports, in a horizontal position, just tight. A mass of 50 lb. is attached at its middle point. If the cord stretches 2 in. for every 1 lb.-wt. direct tension, show that, in a position of equilibrium, the angle θ of deflection from the horizontal is given by the equation

$$12(\tan\theta - \sin\theta) = 1$$

and find θ by help of the tables.

23. Two uniform rods of the same material, AB, BC, rigidly jointed at B so that ABC is a right angle, hang freely in equilibrium from a fixed point at A. If AB makes an angle θ with the vertical, and $AB = a$, $BC = b$, prove that

$$\tan\theta = \frac{b^2}{a^2 + 2ab}.$$

24. A uniform rod, 2 ft. in length, is placed with one end in a smooth hemispherical bowl with its axis vertical. The radius of the bowl being $\frac{1}{2}\sqrt{3}$ ft., find the length of the rod projecting over the edge when it is in equilibrium.

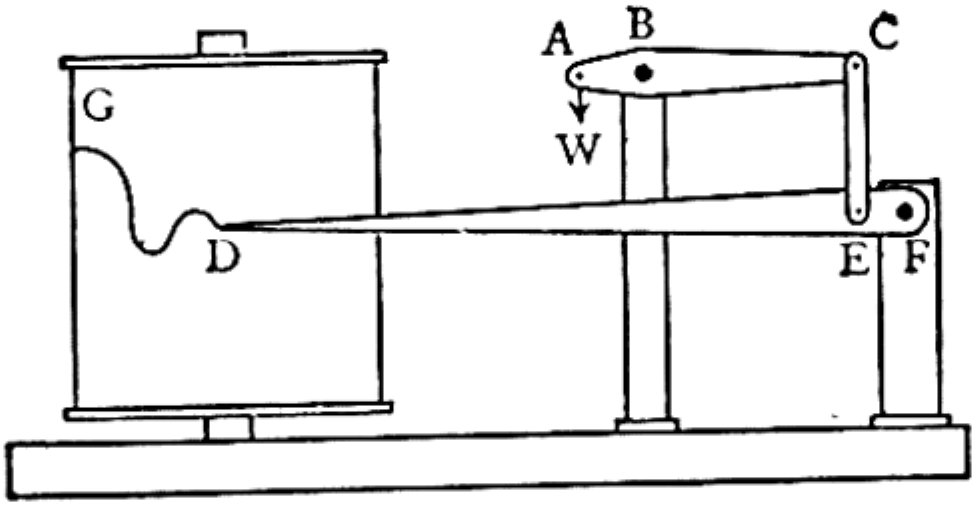
25. The lower ends of two fixed smooth inclined planes intersect in a horizontal line, their inclinations to the horizontal being $\tan^{-1}(\frac{4}{3})$ and $\tan^{-1}(\frac{12}{5})$ respectively. If a rod, length 42 in., mass $1\frac{3}{4}$ lb., can rest horizontally with its ends A and B on the planes respectively, in a plane perpendicular to their line of intersection, find the distance of the centre of gravity of the rod from A, and the reactions at A and B.

SECTION D

1. A body, mass 10 lb., is just moved over a horizontal table by a force of P lb.-wt. The coefficient of friction between body and table being $\frac{1}{2}$, find the value of P when the force acts at inclinations to the horizontal of 0°, 10°, 20°, . . . , 80°. Hence estimate the inclination for which P would be least.

2. A mass of 16 lb. just slips on an inclined plane when the inclination of the plane is 30°. If the inclination is reduced to 20°, what force will just move the mass (*a*) up. (*b*) down, the plane?

3. The figure shows the lever mechanism of a common type of recording barometer. Levers AC and DF are freely pivoted on horizontal axes at B and F respectively and are connected together by the link CE. Movements of the point A are magnified by the levers and are recorded on a slowly rotating drum G by means of a pen attached to the lever FD at D. If the weights of the levers and link are balanced by a weight suspended from A, find the greatest variation which may be made in this weight without causing any movement of the pen when its pressure on the drum is 0·001 lb.-wt. and the coefficient of friction between the pen and the drum is 0·4, all other friction being negligible. AB = 0·5 in., BC = 2 in., DE = 6 in., and EF = 0·5 in.



4. AB is a uniform ladder, length 37 ft., weight W, resting at A on the ground, where $\mu = \frac{1}{2}$, and at B against a vertical wall, where $\mu = \frac{5}{12}$. The distance of A from the wall is 12 ft. A horizontal force is applied at A to move the ladder towards the wall; find its value in terms of W.

5. AB is a vertical post fixed into the ground at A. AC is a uniform bar, length a, mass W, hinged at A, with the end C connected to a point D of AB by a rope. If the angles CAB and CDA are α, β, find the tension in DC. If α is kept fixed, show that the tension may be made a minimum by adjusting the height of D, and find the minimum value.

6. A circular cylinder, diameter $2a$, has its centre of gravity G at a distance $\frac{a}{2}$ from the centre C of the axis and equidistant from the plane ends. The cylinder rests in equilibrium with its axis horizontal on a rough inclined plane, the line CG being also horizontal. Find the inclination of the plane to the horizontal and the least permissible value of the coefficient of friction.

7. A uniform rod, weight W, has one end attached to a light string of equal length whose other end is attached to a point in a rough vertical wall. The rod rests in limiting equilibrium, at an angle θ to the vertical, with its free end resting against the wall. Find the normal and frictional reactions at the lower end of the rod. If $\theta = 75°$, show that the coefficient of friction between rod and wall is 0·8 very nearly.

8. A rough circular cylinder, diameter d, is fixed on an equally rough horizontal plane and a uniform rod, length $2a$, rests tangentially against the cylinder in a vertical plane, which is perpendicular to the axis of the cylinder, one end of the rod being on the rough plane. If the friction is limiting at both ends of the rod, when the rod is inclined at an angle of 30° to the vertical, prove that the angle of friction is $\frac{1}{2}\sin^{-1}\left(\frac{a}{d}\right)$.

9. Particles of mass 2 lb., 1 lb., are placed on the equally rough slopes of a double inclined plane, whose angles of inclination are respectively 60° and 30°, and are connected by a light string passing over a small smooth pulley at the common vertex of the planes. If the heavier particle is on the point of slipping downwards, show that the coefficient of friction is $5\sqrt{3} - 8$.

10. A rod, length a, of negligible mass, rests horizontally between two rough planes, each inclined at 45° to the horizontal, with a particle of mass W placed at the centre of the rod. The coefficient of friction between rod and each plane is 0·5. How far can the particle be gradually moved along the rod before slipping between rod and planes begins?

11. The least force that will just move a body up a rough inclined plane (coefficient of friction μ) is twice the least force that will just prevent the body sliding down the plane. Show that the inclination of the plane to the horizontal is $\tan^{-1} 3\mu$.

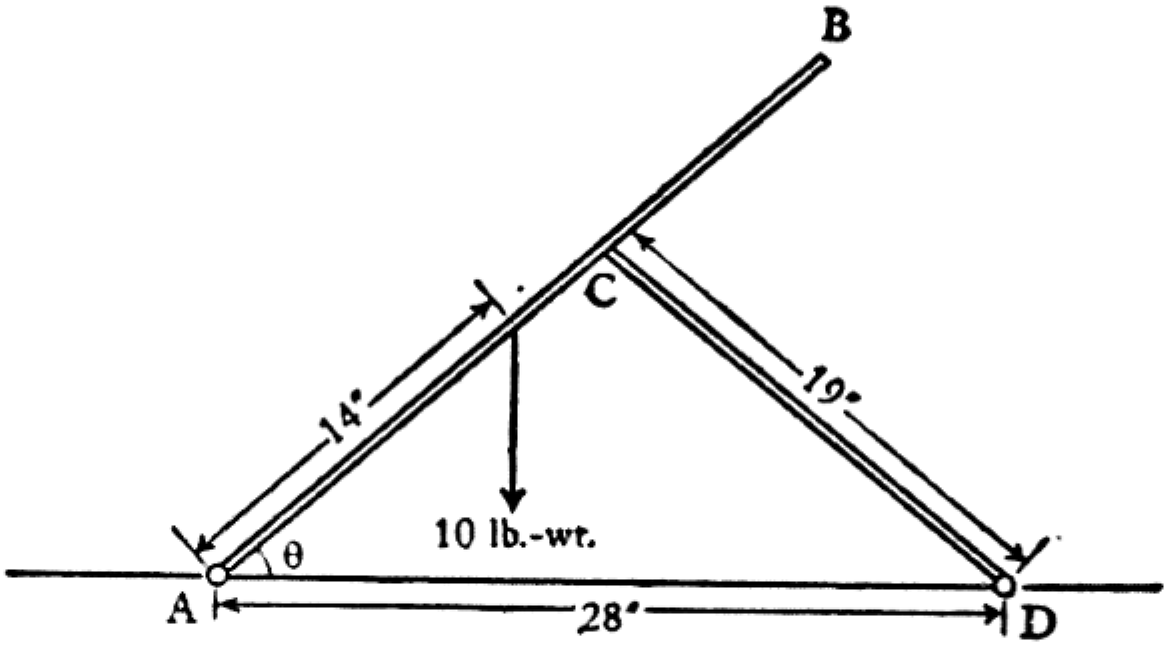
12. A mass of 10 lb., resting on a rough horizontal plane, is pulled by two horizontal strings which include an angle of 150°. In one string there is a constant tension of 3 lb.-wt. The tension in the other is gradually increased until, when its value is P, the mass begins to move. If the coefficient of friction is $\frac{1}{2}$, find the value of P and the direction in which the mass moves, and show this direction clearly in a figure.

13. One face of a uniform cube rests on a rough inclined plane with one pair of edges in the direction of greatest slope; a rope is attached to the mid-point of the uppermost edge of the cube, and runs parallel to the plane in the upward direction of greatest slope. Show that, if the tension in the rope is gradually increased, then the cube will slide or topple over according as the slope of the plane is less than or greater than $\tan^{-1}(1 - 2\mu)$, where μ is the coefficient of friction.

14. A uniform square board ABCD, side a, is held with

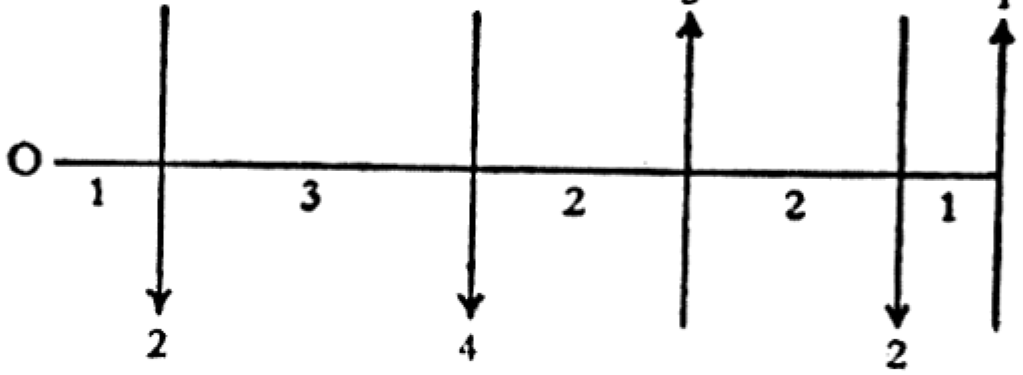
AB in contact with a vertical wall, its plane being perpendicular to the wall, by means of a string attached to a point E of the wall, vertically above B, and to a point P of BC. If BE $= a$, and μ is the coefficient of friction between board and wall, find the limiting values of BP.

15. A trap-door AB (see figure), mass 10 lb., freely hinged at A, is supported at an angle θ to the horizontal by a light prop CD hinged at D and free to move in a vertical plane. If the coefficient of friction between the prop and the door is 0·4, find the least value of the angle θ for equilibrium to be possible and show that the thrust in the prop in this position is 8 lb.-wt.



SECTION E

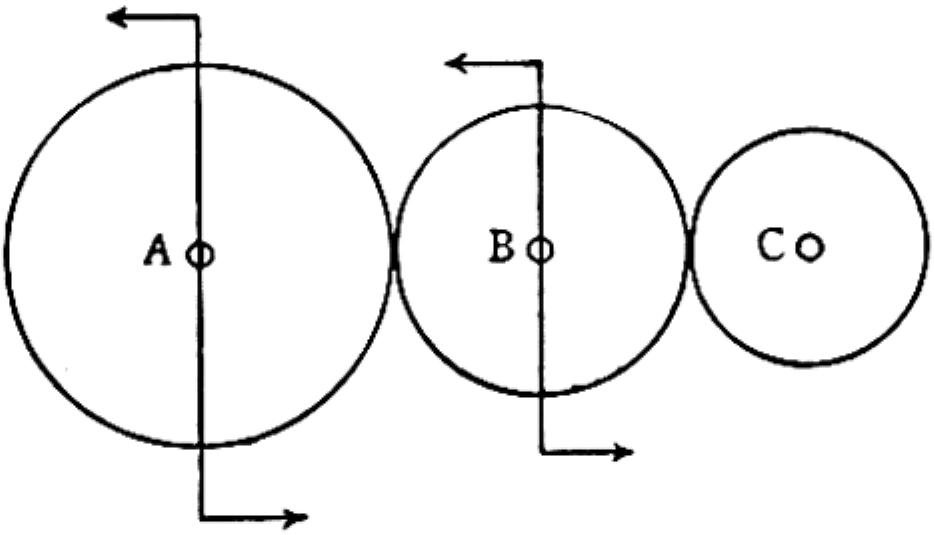
1. The figure represents five parallel forces, expressed in lb.-wt., the distances being in feet. Find the magnitude of their resultant and the distance from O of its line of action. If each of the forces of 2 lb.-wt. is replaced by a



force P lb.-wt., find P so that the system reduces to a couple, and find the moment of this couple.

2. A uniform straight rod AB, length 8 ft., mass 6 lb., is kept at rest in a horizontal position by a vertical force of 12 lb.-wt. at A, a vertical force of 6 lb.-wt. at B, and a couple. Find the moment of the couple. If the couple is exerted by vertical forces at the points of trisection of the rod, draw a figure showing all the forces acting on the rod.

3. Three spur wheels, A, B and C, have 42, 30 and 16 teeth respectively and gear with one another as shown in the figure. Two counter-clockwise couples of 49 and 25 lb.-ft. are applied to the shafts of A and B respectively.



Find the couple that must be applied to the shaft C in order that equilibrium may be maintained. In what direction must this couple act? The diameters of the spur wheels are proportional to the number of teeth on each.

4. ABC is an equilateral triangle, side 2 ft., and AD is the perpendicular from A to BC. Forces of 4, 6, 2, $2\sqrt{3}$ lb.-wt. act along BC, CA, AB, AD respectively. Show that their resultant is a couple and find its magnitude. If the force along AD is increased to $3\sqrt{3}$ lb.-wt., calculate where the resultant of the new system of forces meets BC, or BC produced.

5. ABCDEF is a regular hexagon, side *a*, and forces P, 2P, 3P, 4P, 5P, 6P act respectively from A to B, B to C, C to D, D to E, E to F, and F to A. Find completely the force that will make equilibrium with them. If a further

force acts at the centre of the hexagon so that all seven forces are equivalent to a couple, find this force and the moment of the couple.

6. ABCD is a square, side 1 ft. Forces 2 lb.-wt., 4 lb.-wt., 6 lb.-wt., 17 lb.-wt., $\sqrt{2}$ lb.-wt. act respectively along AD, DC, CB, BA, DB. Find the resultant and the distance of the middle point of DB from its line of action.

7. Forces of P, 4P, 2P, 6P act along the sides AB, BC, CD, DA of a square ABCD of side a. Find the magnitude of their resultant, and find where its line of action meets AD (produced if necessary).

8. ABCD is a rectangle in which AB = 6 ft., BC = 4 ft. E is the mid-point of CD. Forces of 1, 2, 6, 10, 5 lb.-wt. act along AB, BC, CD, DA, AE respectively. Find their resultant and the position of the points of intersection of its line of action with AB and AD.

9. Draw the triangle ABC, having a right angle at C; let BC be 1·5 in. and CA 2·5 in. long. Suppose that a force of 8 units acts from C to A, and one of 5 units from A to B, and one of 10 units from C to B; find the line along which the resultant of these forces acts. Assuming that $\sqrt{34} = 5{\cdot}83$, show that if CB, CA be taken as axes of x and y the equation of the line is

$$1082x - 3665y + 1875 = 0.$$

10. Forces of magnitudes λa, μb, ηc act along the sides BC, CA, AB of a triangle; show that the line of action of their resultant divides BC externally in the ratio $\mu : \eta$ and that the magnitude R of the resultant is given by $R^2 = (\lambda - \mu)(\lambda - \eta)a^2 + (\mu - \eta)(\mu - \lambda)b^2 + (\eta - \lambda)(\eta - \mu)c^2$.

11. In a plane quadrilateral ABCD, AB = 2 in., BC = $2\sqrt{3}$ in., CD = 4 in., angle ABC = 150°, angle BCD = 90°. Forces of magnitudes 4, $\sqrt{3}$ and 5 lb.-wt. respectively act along AB, BC and CD. Find the magnitude of the resultant force, and show that it acts in a direction parallel to AD. Find the distance of its line of action from A.

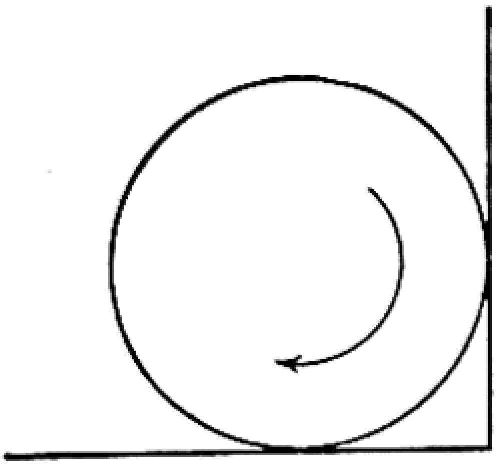
12. Forces 2, 2, 3, 2 units act along the sides AB, CD, ED, EF respectively of a regular hexagon ABCDEF in the

direction indicated by the order of the letters. Find the magnitude of the resultant and prove that it acts along AB.

13. Forces 5P, 3P, 4P act along the lines having, respectively, the equations $3x + 4y = 12a$, $x = 4a$, $y = 3a$, the sense of each force being such that it has an anticlockwise moment about the origin. Find the magnitude of the resultant and the equation of its line of action.

14. OABC is a square, side 1 ft. Forces of 6, 9, 8, 4 lb.-wt. act along OA, AB, CB, CO respectively. Find the distances from O of the points in which their resultant cuts OA and OC. If P is a point about which the sum of the moments of the forces is zero, and (x, y) are the co-ordinates of P referred to OC and OA as axes, find the relation between x and y.

15. A length of circular shaft rests on a horizontal surface and in contact with a vertical surface as indicated in the figure. Show that, for the given piece of shaft, the least value of the couple required to rotate the shaft in a clockwise direction varies as $\frac{\mu(1 + \mu)}{1 + \mu^2}$, where μ is the co-efficient of friction between the surfaces. What is the magnitude of the couple when the shaft is 4 in. in diameter and weighs 90 lb.-wt., μ being 0·15?



16. A square plate ABCD, side 2 ft., rests on a smooth horizontal slab and is freely movable about a pivot at A. It is acted on by forces 6, 6, $8\sqrt{2}$ lb.-wt. along AD, DC, DB respectively. Find the moment of the couple which will keep it in equilibrium.

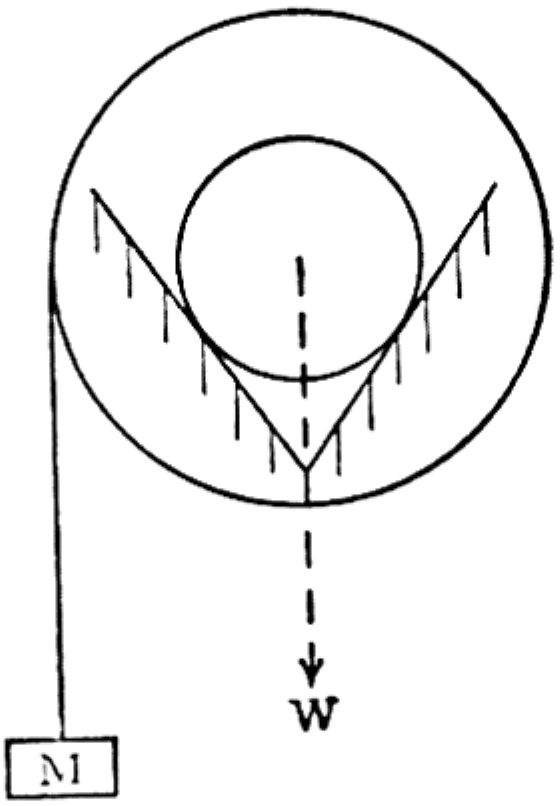
17. A uniform square lamina ABCD, side 2 ft., mass 15 lb., can turn freely in a vertical plane about A, which is fixed, and is kept in equilibrium with the diagonal AC horizontal by means of a pull exerted in a horizontal string through its highest point D together with a couple. Find

the moment of the couple when the magnitude of the reaction at A is 25 lb.-wt.

18. A circular hoop, radius a and weight W, is suspended horizontally by three equally spaced vertical strings, each of length l. The hoop is turned through an angle α about the vertical axis through its centre. Show that the couple required to maintain it in that position is

$$\frac{Wa^2 \sin \alpha}{\sqrt{l^2 - 4a^2 \sin^2 (\alpha / 2)}}.$$

19. The axle of a wheel rests in V-grooves as shown in the figure, and a string, fixed to the rim of the wheel, supports a mass M. If μ is the coefficient of friction, W the mass of the wheel and axle, a the radius of the axle and b the radius of the wheel, show that the least value of M which will cause motion is given by $\frac{\mu W a}{b(1+\mu^2)\sin\theta - \mu a}$, where 2θ is the angle of the groove.



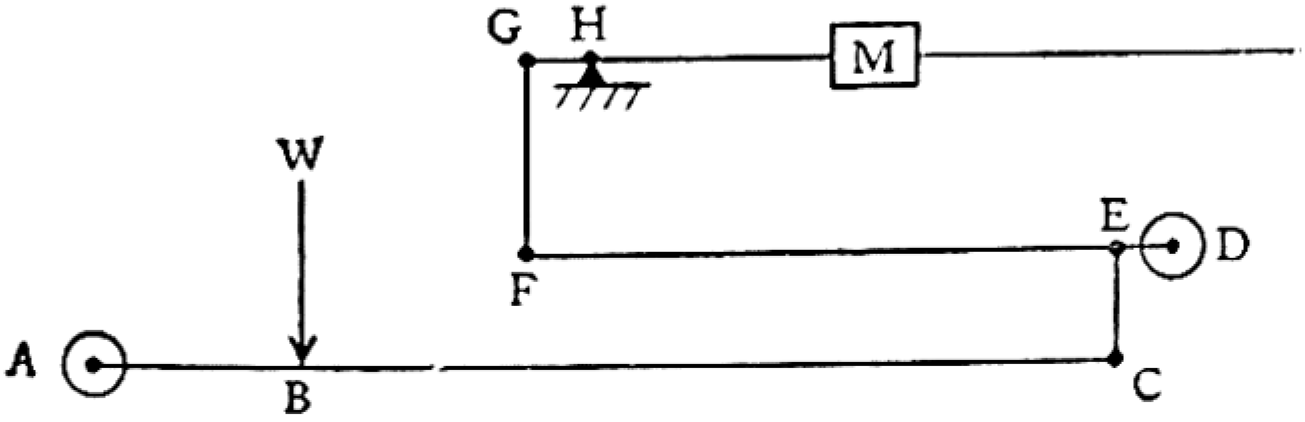
20. If the sums of the moments of a system of coplanar forces about the three points $(-a, 0)$, $(a, 0)$, $(0, a\sqrt{3})$ are r, s, t respectively, show that the magnitude of the resultant force is

$$\frac{1}{a}\sqrt{\frac{r^2 + s^2 + t^2 - rs - st - tr}{3}}.$$

SECTION F

1. The wheels of a bicycle are 30 in. in diameter and turn $3\frac{1}{2}$ times for one complete revolution of the pedals, the arms of which are 6 in. long. The bicycle weighs 2 st. Find the maximum gradient up which a man weighing 10 st. can make the bicycle move by putting all his weight on one pedal in the horizontal position, if a force of 7 lb.-wt. must be applied to the pedal to overcome friction.

2. The figure shows the arrangement of levers in a testing machine. A, H and D are fixed fulcra, and the jockey weight M is 56 lb.-wt. If the system is initially in equilibrium, find how far M will have to be moved to balance an additional 1-ton load at B. Find also the changes of the reactions at A, D and H. AB = 6 in. AC = 30 in. ED = $1\frac{1}{2}$ in. DF = 18 in. GH = $1\frac{1}{2}$ in.



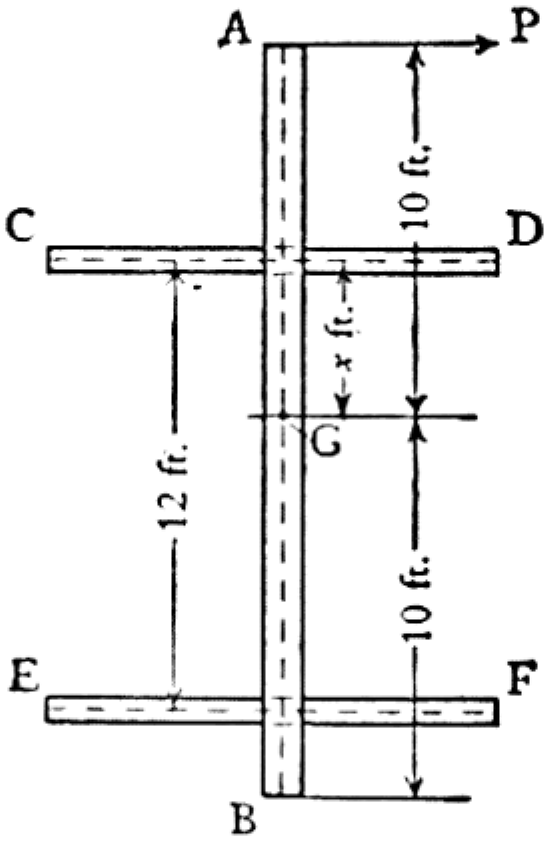
3. Two grooved wheels, radii a, b, ($a > b$), are keyed to the same axle which is rotated by a crank the length of whose arm is c. A light cord passes under a smooth pulley supporting a weight W, and has its ends coiled round two wheels so that when the crank is turned the cord is coiled on to one pulley and off the other. If a force P is applied to the end of the crank to prevent motion, obtain the algebraic relation between P and W, assuming there is no friction. If there is friction and the efficiency of the machine is 40 per cent, find the weight W that can be just raised by a force P = 25 lb.-wt., given that $a = 12$ in., $b = 9$ in., $c = 6$ in.

4. Loads are being raised by a rope which, after passing

over a pulley, is wound on a drum of 6 in. diameter to which is keyed a cogwheel of 84 teeth, which is turned by means of an endless screw operated by a crank handle of 10 in. crank radius. Each turn of the screw moves the cogwheel 1 tooth. Find the velocity ratio of the machine, and, if the efficiency is 40 per cent, find the force which must be applied to the crank handle to raise 1 ton.

5. A weight is raised slowly by a vertical tackle. The lower block, to which the weight is attached, has 2 sheaves and the fixed block has 3 sheaves. The running end and the portions of the rope between the sheaves may all be assumed vertical. Sketch the arrangement and find the velocity ratio. If the work wasted in friction is $\frac{5}{7}$ times the useful work done, find the mechanical advantage and the efficiency. (Neglect the weight of the rope and blocks.)

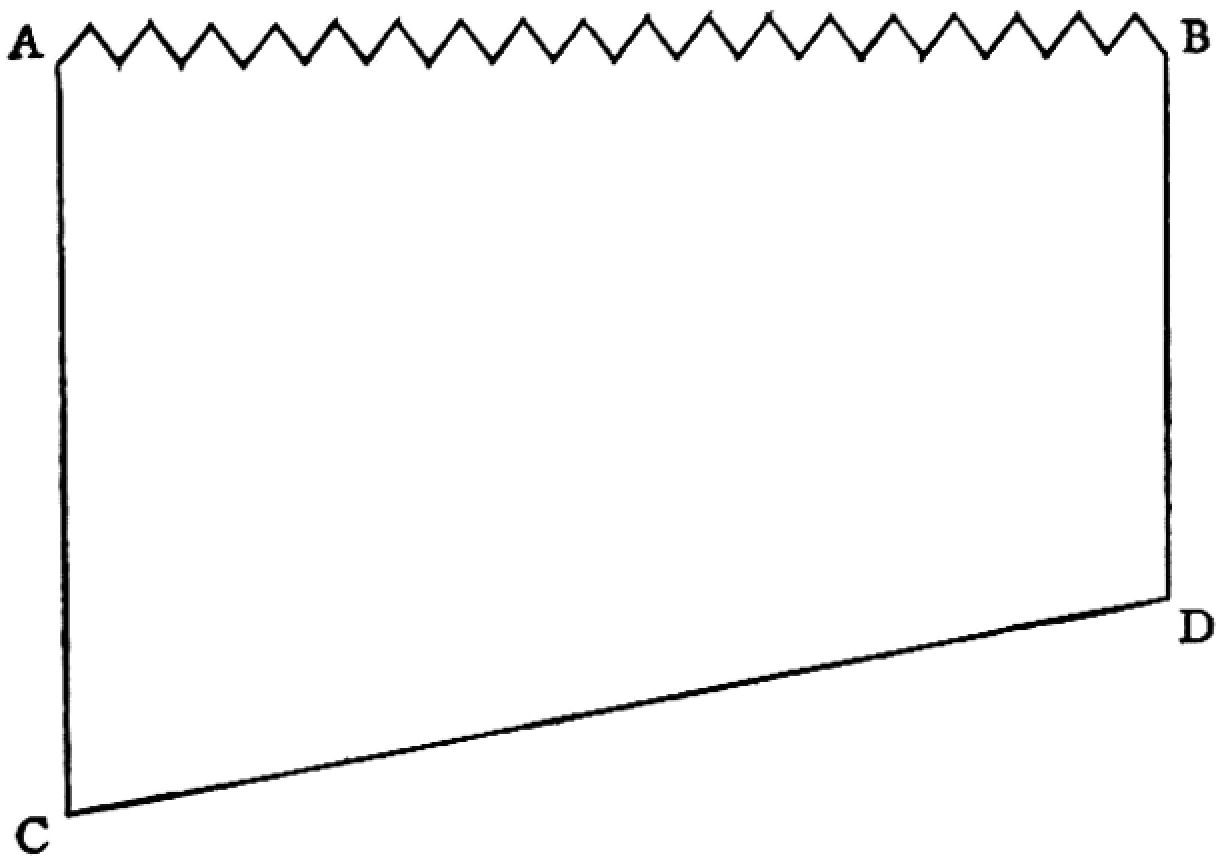
6. A uniform horizontal girder AB, mass 9 tons, lies at right angles to two parallel horizontal rails CD and EF. The centre of gravity of the girder is distant x ft. from the rail CD as shown in plan in the figure. If a horizontal force P is applied at A and just causes both ends of the girder to move, find the distance x and the magnitude of P. (The coefficient of friction between the girder and the rails is 0·01.)



7. A thin rectangular metal strip, length 12 in., of uniform width, is bent into two parts, lengths 9 in. and 3 in. Show that the metal in this form can rest without upsetting with the shorter part in contact with a horizontal plane so long as the angle between the two parts is not less than the acute angle whose cosine is $\frac{7}{9}$ or greater than the obtuse angle whose cosine is $-\frac{1}{9}$.

8. C is the centre of a sphere of uniform material, radius 10 in., which contains two spherical cavities, centres A, B, radii 1 in., 2 in., situated so that $CA = CB = 7{\cdot}5$ in. and $AB = 9$ in. If the sphere rests on a horizontal plane, find the tangent of the angle made by the line AB and the vertical.

9. A portion of a saw is depicted in the figure. If AB is taken as horizontal, then CD is inclined at 9° to the horizontal. $AB = 10$ cm., $BD = 5$ cm., and the perpendicular distance of the tip of each saw tooth from AB is 3 mm. Find the distances of the centre of gravity of the portion from AC and AB.



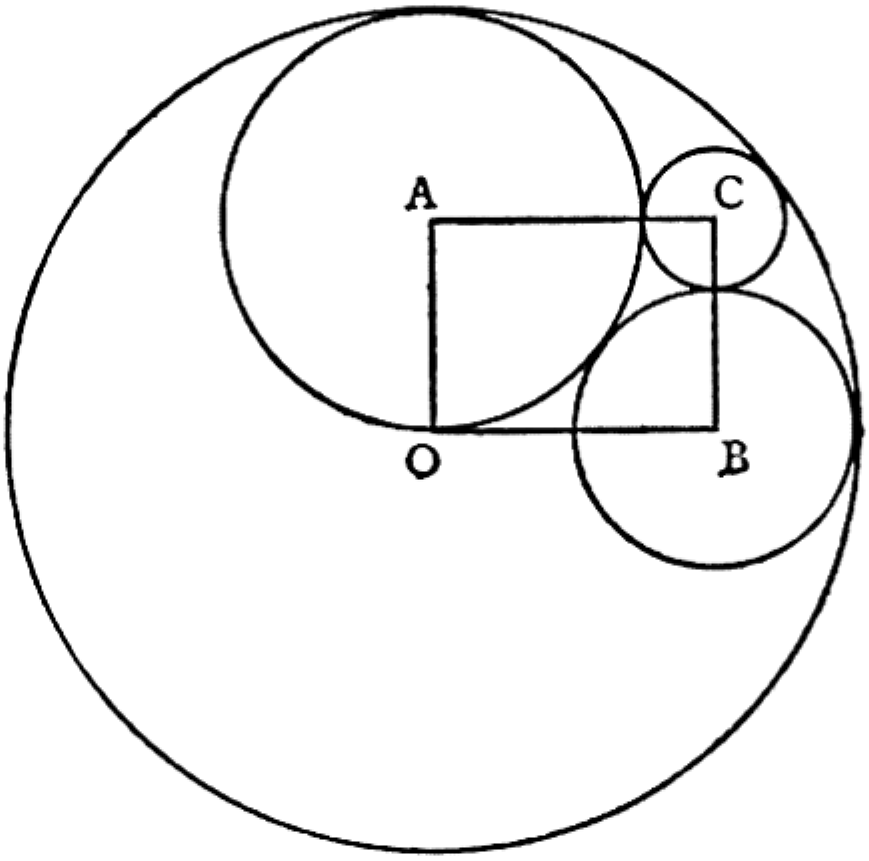
10. From a uniform solid right circular cylinder, length 8 in., is removed the solid right circular cone whose base is one circular face of the cylinder and whose vertex is the centre of the other circular face. Find the distance of the centre of gravity of the remaining solid from this last point. If the conical hole is now filled with substance whose density is twice that of the original solid, find the centre of gravity of the figure so formed.

11. Determine the centre of mass of a uniform solid right cone of height h and semi-vertical angle α. Determine α in order that the centroid of the whole surface (including the base) may coincide with the centre of mass of the volume.

12. A " sand-castle ", turned out of a bucket, has height 9 in., radius of top 3 in., radius of base 5 in. Find the height of its centre of gravity.

13. A uniform solid sphere is divided into two parts by a spherical surface which has its centre on the surface of the solid sphere and has the same radius r as the solid sphere. Find the positions of the mass-centres of the two parts.

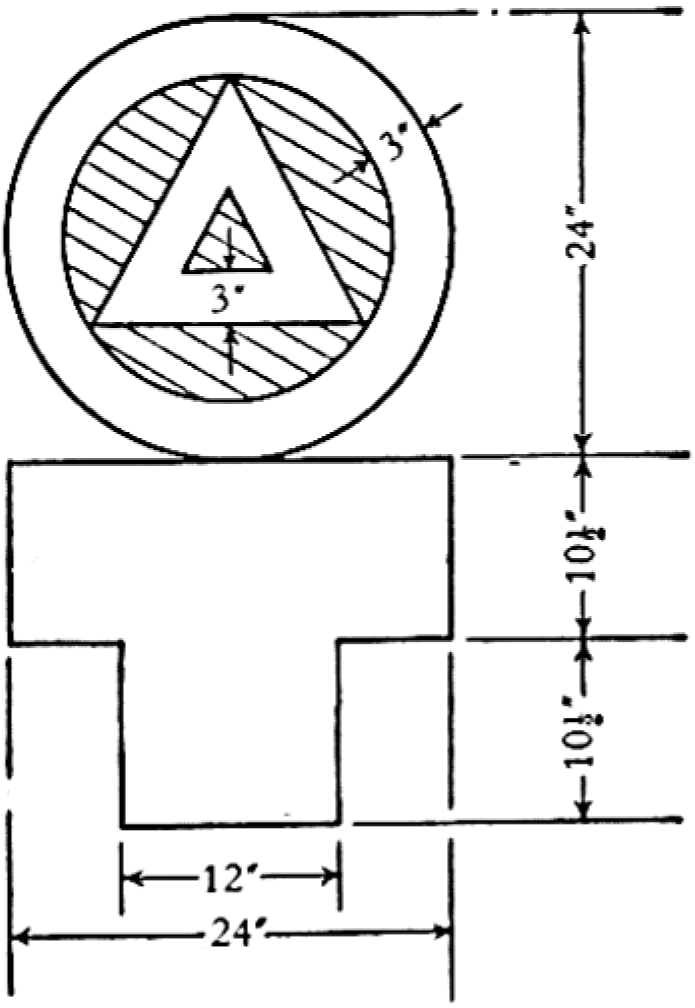
14. A rectangle OACB has sides $OA = 3a$, $OB = 4a$; circles with centres O, A, C, B have radii $6a$, $3a$, a, $2a$. A



uniform lamina has the form obtained by removing from the largest circle the three other circles (see figure). Prove that the reflection in O of the centroid of the lamina divides the line joining O to the mid-point of AC in the ratio 5 : 6.

15. A thin circular plate consists of two parts separated by a diameter; each part is of uniform density, but the density of one part is twice that of the other. Find the centre of gravity of the plate.

16. A road sign has shape and dimensions as shown in the figure, the shaded portions representing spaces. Assuming the material to be of uniform thickness and density, find the distance of its centre of gravity from the bottom edge of the sign.

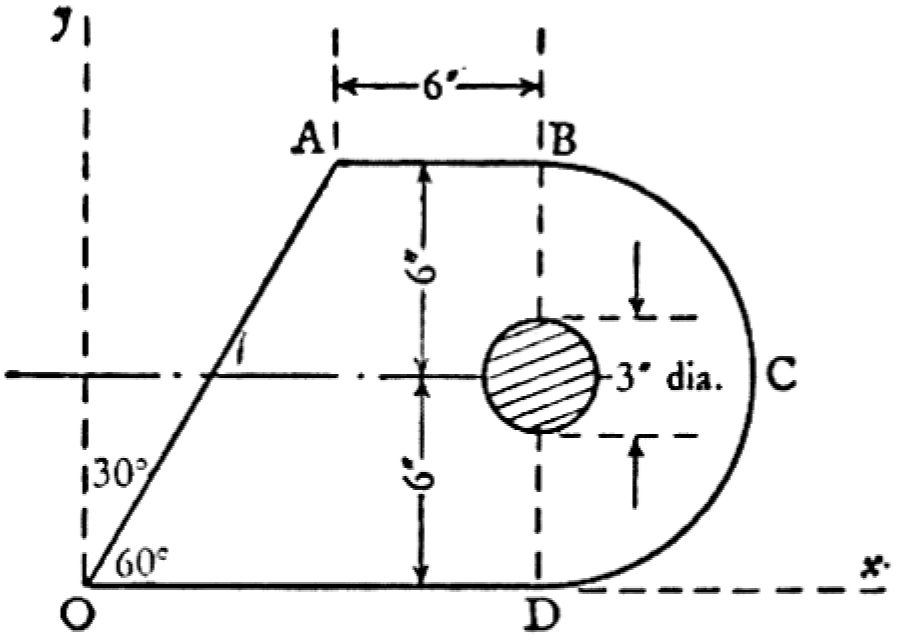


17. A uniform semicircular lamina, bounding diameter AOB (O being the centre), is suspended from B. A portion in the form of a semicircle, having OA as diameter, is removed. Find the inclination of AB to the vertical.

18. A light rectangular card OXRY is loaded with four equal masses, one on each edge and at the ends of two lines APB, CPD drawn on the card parallel to the edges OX, OY respectively. Show that if the loaded card is

suspended by a string from any point of the card, the line of the string will pass through the mid-point of PQ, where Q is the intersection of OR, XY.

19. A sheet of metal of uniform thickness is shaped as shown in the figure. The shaded area represents a circular



hole, diameter 3 in. Find the position of the centre of gravity, stating its distances from Ox and Oy.

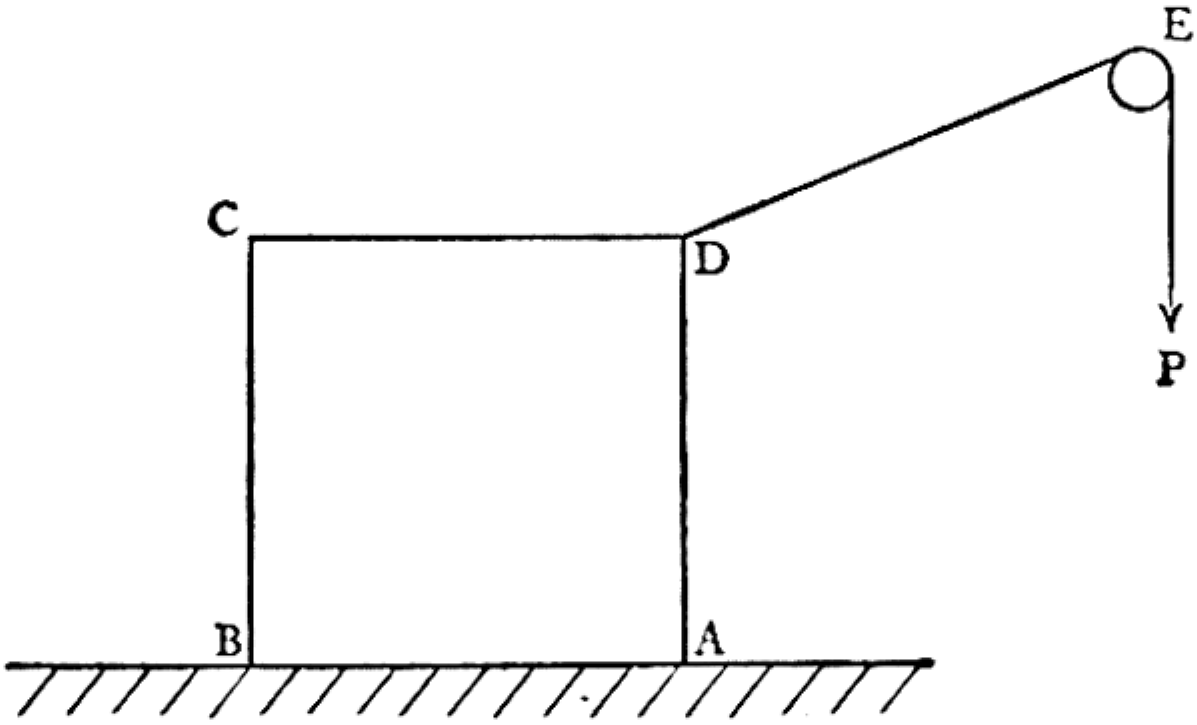
20. If the card of question 18 is uniform and heavy, with mass equal to the sum of the four loading masses, show that the line of the string will always pass through the point of quadrisection of PQ nearer Q.

SECTION G

1. Three equal uniform rods of mass W and length $2a$ are smoothly jointed to form a triangle, which is supported in a vertical plane by two sides resting on two smooth pegs in a horizontal line at distance $\frac{a}{3}$ apart. The other rod is horizontal and uppermost. Find the magnitudes and directions of the reactions at the lower joint and at one of the upper joints.

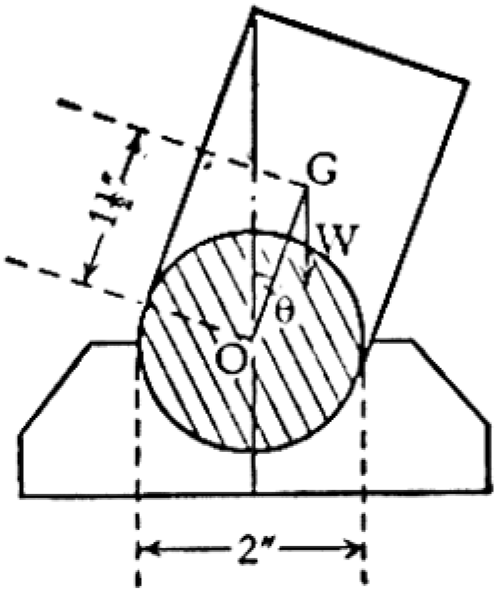
2. Let a square ABCD (see figure) represent a cube with its face AB on a rough horizontal plane, and let the

angle of friction be θ. There is a pulley at a point E over which passes a thread DE, fastened to D and carrying a mass P at the hanging end. DE is inclined at an angle α to the vertical. When P is just on the point of turning the cube round A, and of making it slide simultaneously, show that



$$\operatorname{cotan}\theta + \operatorname{cotan}\alpha = 2.$$

3. A single-throw crankshaft rests in semicircular bearings as shown in the figure, G being the centre of gravity of the crankshaft, and O being the axis of the bearings.



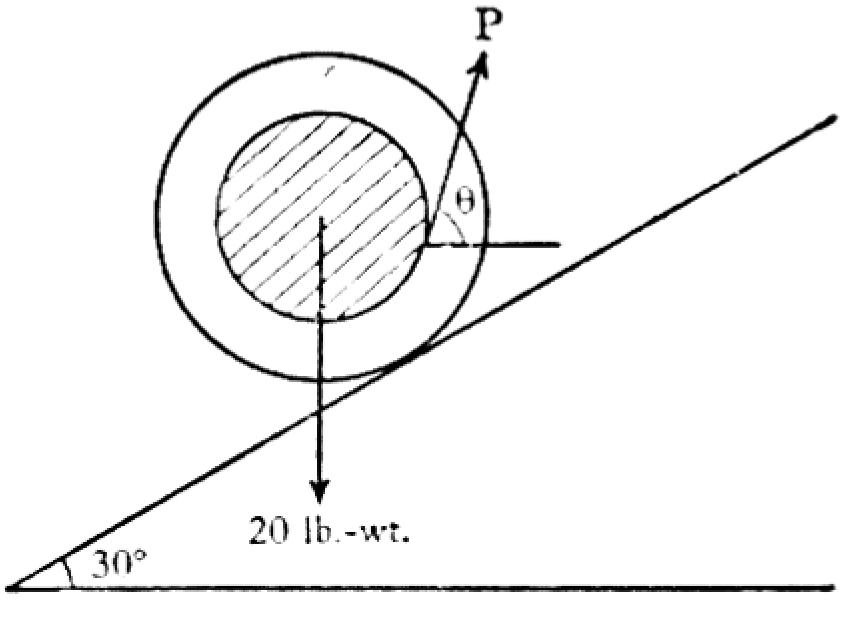
The journals on the crankshaft are 2 in. diameter and the distance OG is 1½ in. The coefficient of friction between the journals and the bearings is 0·5. Show that for equilibrium to be maintained the angle θ which OG makes with the vertical must not exceed $17\frac{1}{2}$ degrees approximately.

4. A smoothly jointed framework is composed of five light rods in the form of a square ABCD stiffened by a diagonal BD. Masses of 19 lb. and 5 lb. are hung from the joints D and C respectively, and the framework is supported by a horizontal knife edge under the mid-point M of the rod AB which is uppermost. Show that in the position of equilibrium the rod CD makes an angle $\tan^{-1}\frac{7}{24}$ with the horizontal, and find the stress in this rod.

5. Two equal uniform rods of length $2a$ are hinged freely together, and are supported symmetrically by two pegs on the same level, at distance $2c$ apart. Show that, if λ is the angle of friction between each peg and rod, the minimum inclination α of each rod to the vertical satisfies the equation

$$a \sin^2\alpha \,.\, \sin(\alpha + \lambda) = c \cos\lambda.$$

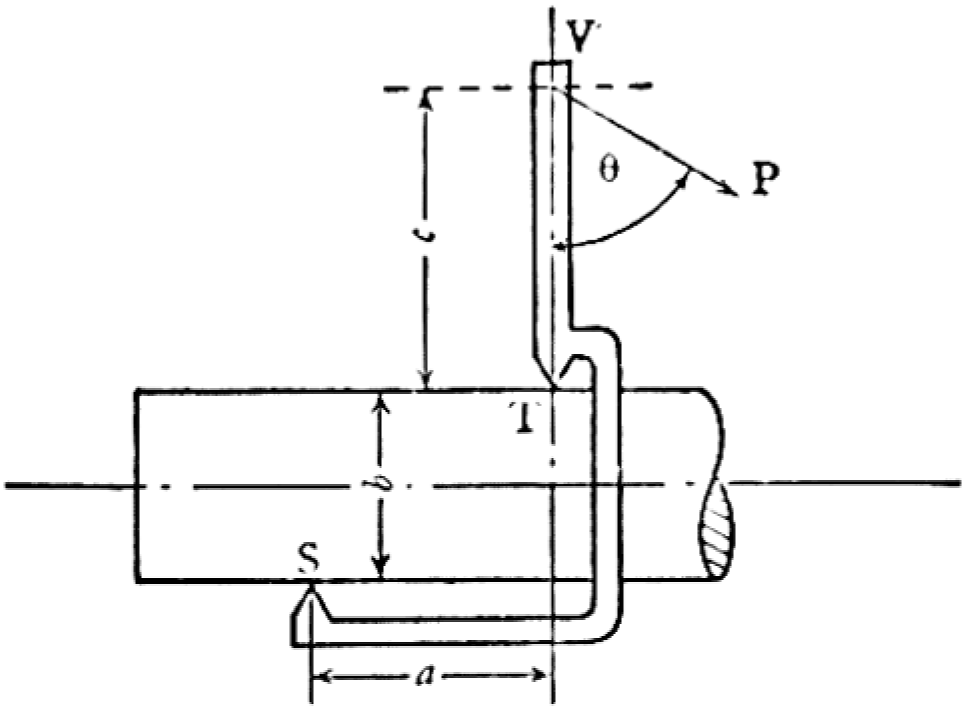
6. A pair of equal wheels are connected by an axle of which the diameter is $\frac{2}{3}$ the diameter of one of the wheels. The total mass is 20 lb. It is held in equilibrium on a rough



plane, inclined at 30° to the horizontal, by a force P applied at the end of a string wrapped around the axle as shown in the figure, which represents a cross-sectional elevation through the centre of gravity. The line of action of P is in the plane of the figure. If θ is the angle made by P with the horizontal, find the value of P when θ is 15°, assuming that the wheels do not skid. Find also the least value of the coefficient of friction between the wheels and the plane.

7. On a horizontal shelf n equal sheets of paper rest in a pile, the coefficient of friction between paper and paper being μ_1, and between paper and shelf $\mu_2 (< \mu_1)$. A gradually increasing pull is applied horizontally to the middle of the edge of the rth sheet counting from the top. Show that if $r \cdot \mu_1 < n \cdot \mu_2$ the r topmost sheets only will ultimately begin to move, but if not, then the whole pile will begin to move.

8. A light clamp consists of a ring with two arms at right angles to each other. The ring surrounds a fixed rod and the arms touch the rod at the points S and T as shown in the figure. A force P is applied at V in the plane of STV. If the clamp is just on the point of sliding to the right, show that $\tan\theta = \frac{\mu(a - \mu b)}{a - \mu(b + 2c)}$, where μ is the coefficient of limiting friction.



9. An isosceles triangular wedge, vertical angle 2θ, is placed on its side on a rough inclined plane, the thin edge being perpendicular to the lines of greatest slope and uppermost. The inclination of the plane being gradually increased, show that equilibrium will be broken first by the wedge toppling over, provided that

$$2\mu > (3\tan\theta + \cot\theta).$$

10. A uniform cube rests on a rough plane inclined at an angle α to the horizontal, with one set of edges parallel to the lines of greatest slope. A gradually increasing horizontal force is applied at the mid-point of the uppermost edge, in a direction perpendicular to the edge and tending to push the cube down the plane. If λ is the angle of friction, show that the cube will eventually topple over if

$$\tan\left(\frac{\pi}{4} - \alpha\right) < 2\tan(\lambda - \alpha).$$

11. AOB, COD are two perpendicular diameters of a uniform circular lamina of mass 4W. The lamina is fixed at the point A so that it can rotate freely about A in its own plane, which is vertical, and masses W, 2W, 3W are attached at the points C, B, D respectively. Find the inclination of OA to the vertical in the position of equilibrium.

12. A uniform heavy chain of length a rests partly along a line of greatest slope of a rough plane inclined at an angle α to the horizontal, and partly hanging over the edge at the top of the plane. If the coefficient of friction between chain and plane is μ, show that the maximum length which can hang with the chain in equilibrium is

$$a(\sin\alpha + \mu\cos\alpha)/(1 + \sin\alpha + \mu\cos\alpha).$$

Find the minimum length.

13. A uniform heavy rod of length $2a \,.\, \cos\beta$ is inside a rough spherical bowl of radius a. Show that, if the angle of friction at each end of the rod is λ, the rod can rest freely with one end at the lowest point of the bowl provided that

$$\cos(\beta + 2\lambda) \leqslant \cos\beta \,.\, \cos 2\beta.$$

14. A uniform beam, length $2a$, is placed in a fixed smooth hemispherical bowl of radius r, $r < a$; show that in the position of equilibrium the angle θ which the beam makes with the vertical is given by

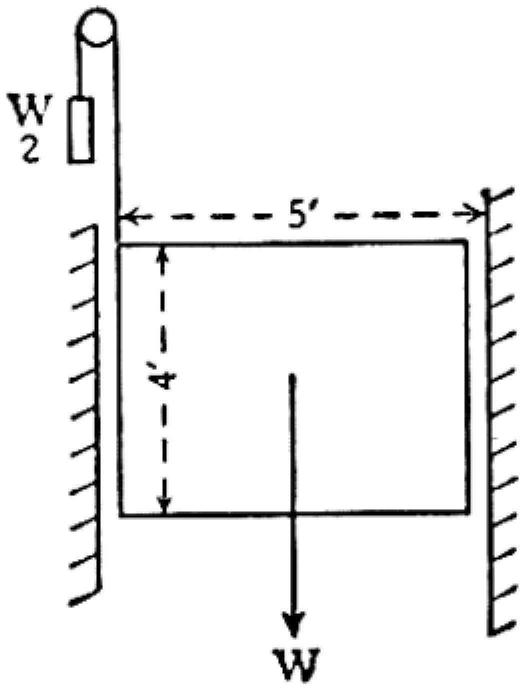
$$\sin^2\theta - \frac{1}{4}\frac{a}{r}\sin\theta - \frac{1}{2} = 0.$$

15. The two sides of a stepladder have the same length but unequal masses W, W_1 (where $W < W_1$), and are freely jointed at the top. The steps are gradually opened out while standing on a rough horizontal floor, the coefficient of friction between floor and ladder being μ. Show that slipping occurs when the angle θ between the sides is given by

$$\tan\frac{\theta}{2} = \mu\,\frac{3W + W_1}{W + W_1}.$$

16. Two equally rough planes A, B, whose line of intersection is horizontal, are perpendicular to each other and make angles α, $\frac{\pi}{2} - \alpha$ with the horizontal. A uniform rod rests with its ends in contact with the planes and is perpendicular to their line of intersection. If λ is the angle of friction, find the greatest angle which the rod can make with the plane B.

17. A loose-fitting window is normally balanced by two vertical sash cords, passing over pulleys and carrying two equal weights. Friction of the pulleys is represented by a force equal to one-tenth of the weight hanging over the pulley. The breadth and depth of the window are 5 and 4 ft. respectively. What is the least coefficient of friction between the window and its frame in order that one weight alone (as shown in the figure) equal to half the weight of the window shall be sufficient to stop it from falling?



18. Two heavy rings, each of mass W, slide on a rough horizontal rod (coefficient of friction μ). They are connected by a string, b ft. long, to the centre of which is fastened a third mass, W. Show that in the limiting position, when both rings are on the point of slipping, the distance between the rings is

$$\frac{3\mu}{\sqrt{(1+9\mu^2)}}\,b.$$

19. The cross-section of a prism is a regular polygon of 24 sides, and the prism can be turned about its axis, which is horizontal. On the uppermost face, which is horizontal, is placed a block whose coefficient of friction is 0·5, and on each of the two adjacent faces are placed blocks with coefficient of friction 0·6. The prism is then turned slowly about its axis. Find which of the blocks is the first to slide. Through what angle will the prism then have been rotated?

20. A circular cylinder, mass W, is placed with axis horizontal on a smooth plane, which is inclined at an angle a to the horizontal; a string is attached to a point in the plane, and after passing over the cylinder in the vertical plane which contains a line of greatest slope of the inclined plane, supports a mass P. If the straight portions of the string are horizontal and vertical, show that for equilibrium

$$\frac{P}{W}=\frac{\tan a}{1-\tan a}.$$

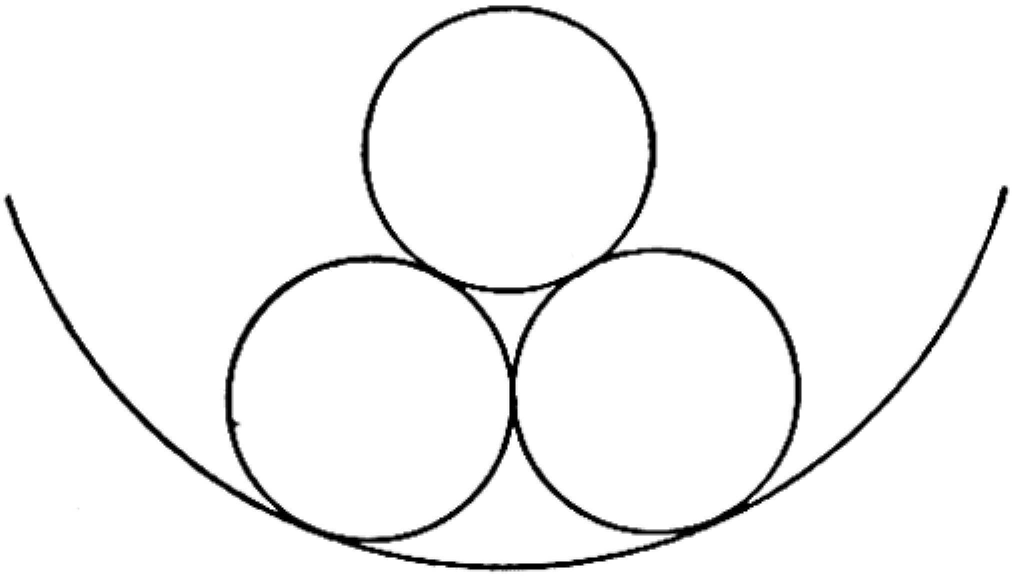
21. A light rope ABCDE is fixed at the ends. It carries a mass of 8 lb. at B and one of 6 lb. at D. At C a force F is applied (in the plane of the segments of the rope) such that AB slopes down at 45°, BC is horizontal, and CD, DE slope upwards at 30° and 60° respectively. Find the tension in all the segments of the rope, and the magnitude and direction of F.

22. Two masses W and W_1, joined by a tight string, rest on a rough horizontal board. The coefficients of friction between masses and board are μ and μ_1, where μ_1 is the

greater. The board is gradually tilted upwards about an edge in front of W and perpendicular to the string. Show that the bodies will be about to slip when the board has been turned through an angle α given by

$$\tan\alpha = \frac{\mu W + \mu_1 W_1}{W + W_1}.$$

23. A smooth hollow open semicircular cylinder of radius R whose axis is horizontal contains two equal circular cylinders of radius a, where $a < R/2$. These rest in equilibrium, and a third equal cylinder is placed on top of the other two, the axes of all the cylinders being parallel and horizontal. Show that the two lower cylinders will separate if



$$R > a(1 + 2\sqrt{7}).$$

24. A uniform plank, length $2a$, has one end resting on rough ground at O and is being slowly lowered to the ground by a rope fastened to the plank at a distance $x (< a)$ from the other end and paid out from a point at distance $2a$ vertically above O. Show that the plank will be on the point of slipping at O when it makes an angle θ with the horizontal given by

$$\cos(\theta + \lambda) = \frac{2(a - x)\sin\lambda}{2a - x},$$

where λ is the angle of friction.

25. A heavy block on a rough horizontal plane just slips when pulled with a force of 507 lb.-wt. by a rope inclined

at $\tan^{-1}(\frac{5}{12})$ to the horizontal. When the inclination is increased to $\tan^{-1}(\frac{3}{4})$ a force of 533 lb.-wt. is necessary to cause the block to slip. Find the mass of the block and the force necessary to move it when the inclination of the rope is $\tan^{-1}(\frac{4}{3})$.

26. A particle, mass W, rests against the circumference of a circular plate, whose plane is vertical; a cord attached to the particle passes over a pulley, placed vertically above the highest point of the circle at a distance from the circle equal to the radius, and carries a mass P; show that the particle will rest at an angular distance θ from the highest point of the circle, where

$$\cos\theta = \frac{5}{4} - \frac{P^2}{W^2}.$$

Prove also that the pressure on the circle is independent of the magnitude of P.

27. Two light rods AB, BC, each of length $2a$, rigidly joined at right angles to each other at B, are placed astride a fixed rough horizontal circular cylinder, radius a, so that they are equally inclined to the vertical. Weights w, $W (W > w)$ are hung from A and C respectively, their values being such that slipping is about to take place. Prove that

$$\frac{W}{w} = \frac{1 + \mu + \mu^2}{1 - \mu + \mu^2},$$

where μ is the coefficient of limiting friction.

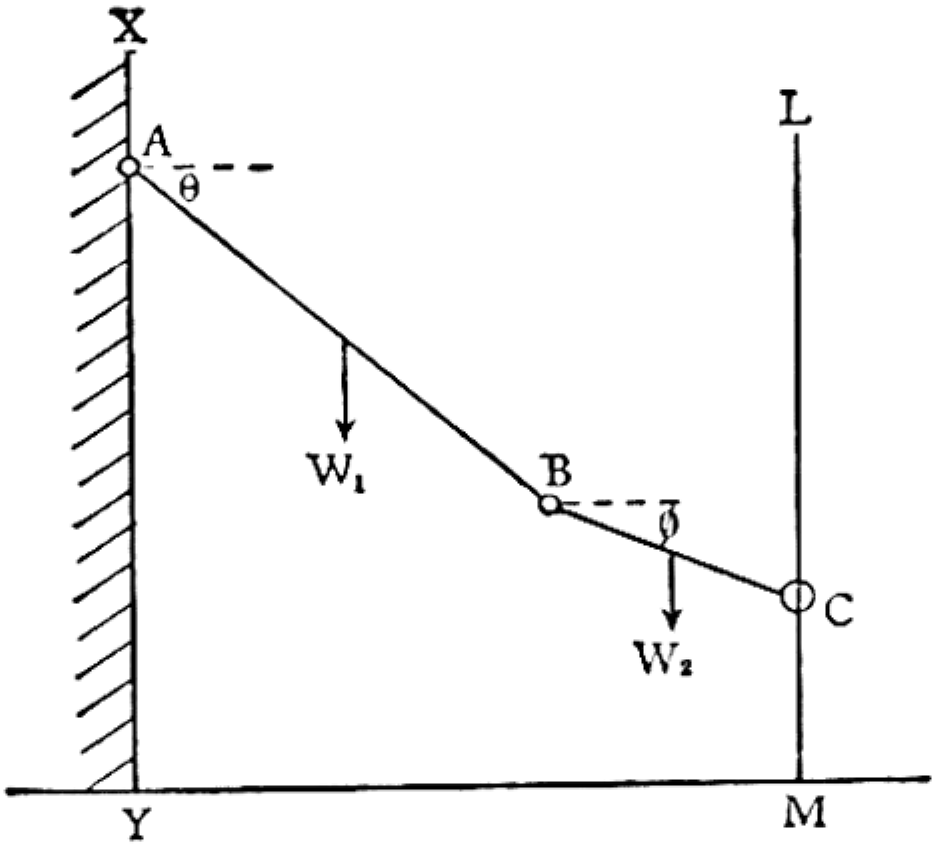
28. Masses W, nW, W are attached to a light string OABCD, length $4a$, at the points of quadrisection A, B, C respectively. O is fastened to a point of a thin horizontal rough fixed rod, D is attached to a small light ring which can slide on the rod. If the ring is in limiting equilibrium, the coefficient of friction is $1/\sqrt{3}$, and BC makes an angle 45° with the vertical, prove that $OD = a(1 + \sqrt{2})$ and find the value of n.

29. Forces of equal magnitude F act at opposite ends of the diameter $y = 0$ of the circular disc $x^2 + y^2 = a^2$, and

in its plane. The force at $(-a, 0)$ is parallel to the y-axis; the angle between the forces is θ. Find the equation of the line of action of the resultant of the two forces and so show that the disc cannot be kept in equilibrium by a third force, applied at some point of the disc, if θ is greater than

$$\pi - \cos^{-1}(\sqrt{5} - 2).$$

30. In the diagram, AB is a uniform rod of weight W_1, freely jointed at A to a vertical wall XY and at B to a uniform rod BC of weight W_2. The end C of the rod BC carries a small ring which can slide freely on a fixed vertical smooth rod LM. The reaction on the ring at C is R and the inclinations of AB, BC to the horizontal are θ, ϕ; prove that, when the rods AB, BC are in equilibrium with A, B, C in one vertical plane,



$$2R = (W_1 + 2W_2)\cot\theta = W_2\cot\phi.$$

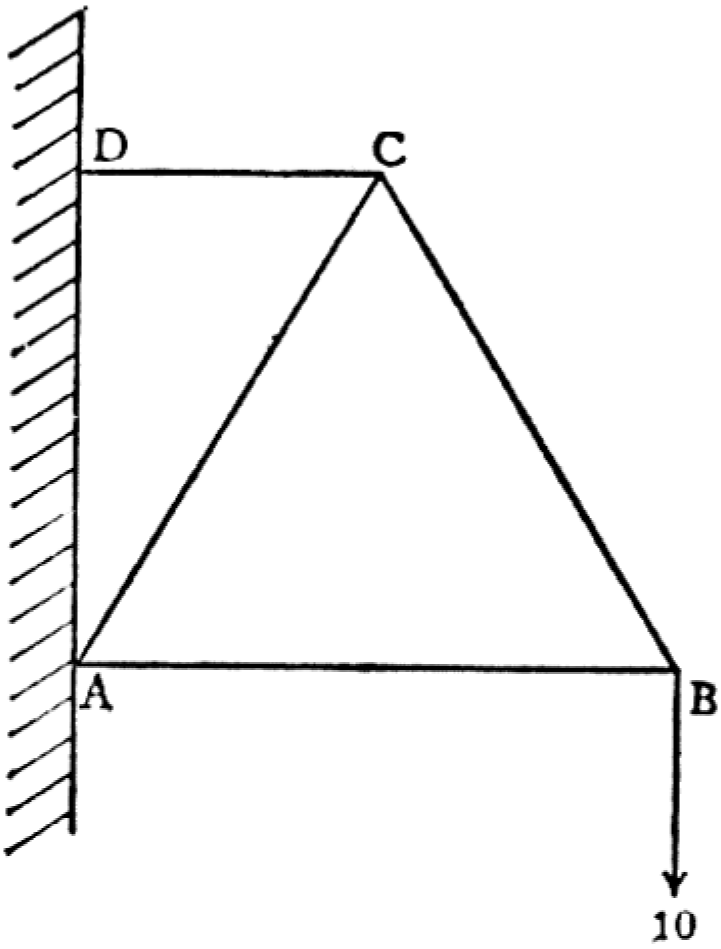
The angle between the horizontal and the direction of the reaction of AB on BC is α; prove that

$$\tan\alpha = 2\tan\phi.$$

SECTION H

1. A rope, length 15 ft., fastened to two points P, Q, carries a mass W which can slide along the rope. The height of Q above P is 3 ft., the horizontal distance of Q from P is 12 ft. Find, graphically, the depth of W below P in the equilibrium position and the tension in the rope.

2. Three equal light rods, AB, BC, CA, jointed at B and C, are fastened to a vertical wall at A (see figure). A fourth rod is jointed to the others at C and fixed to the wall at D, above A. If DC and AB are horizontal, and a load of 10 cwt. is hung from B, find the stresses in the rods.

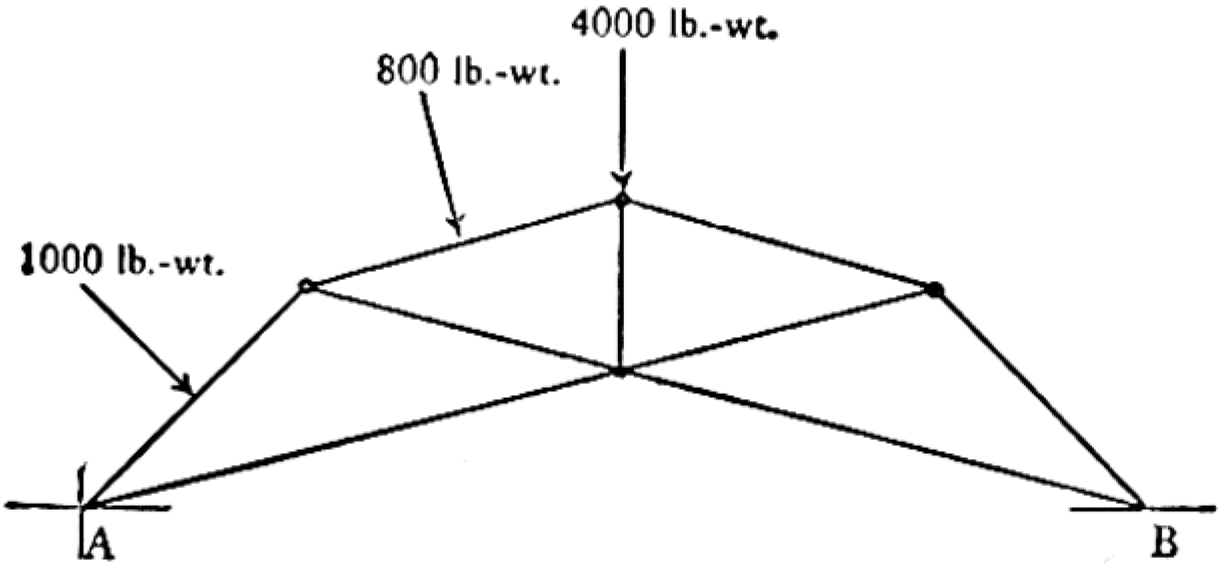


3. AB, BC, CD, DA are four equal uniform rods, each of mass W, freely jointed at their extremities, suspended from the joint A and kept in the form of a square by means of a light string joining A and C. Find the reactions at the joints B and D, and the tension in the string.

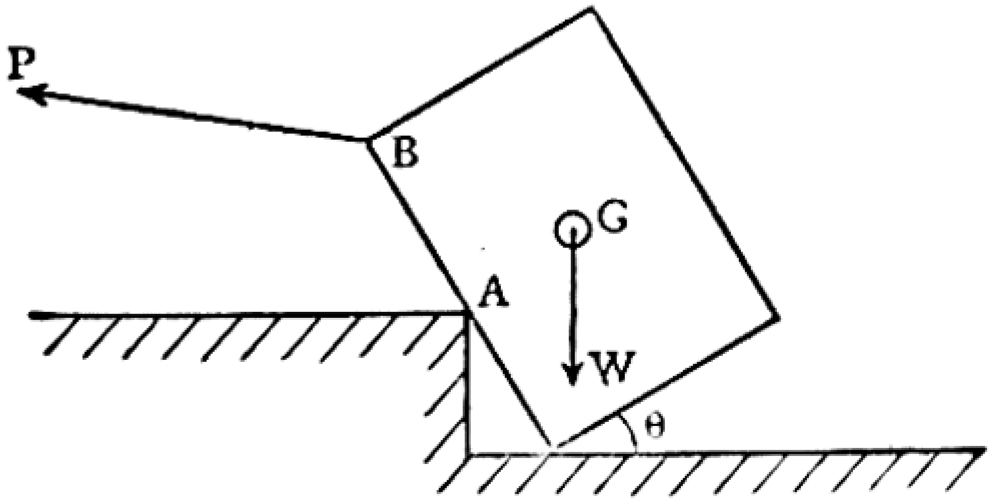
4. Four light rods, AB, CD, BC, AD, lengths 4, 10, 5, 5 ft. respectively, freely jointed at A, B, C, D, form a trapezium in a vertical plane, AB and CD being horizontal,

the former uppermost. The whole is placed on smooth horizontal supports at C and D, and a mass of 200 lb. is placed at the mid-point of AB. Find, graphically, the stresses in the rods.

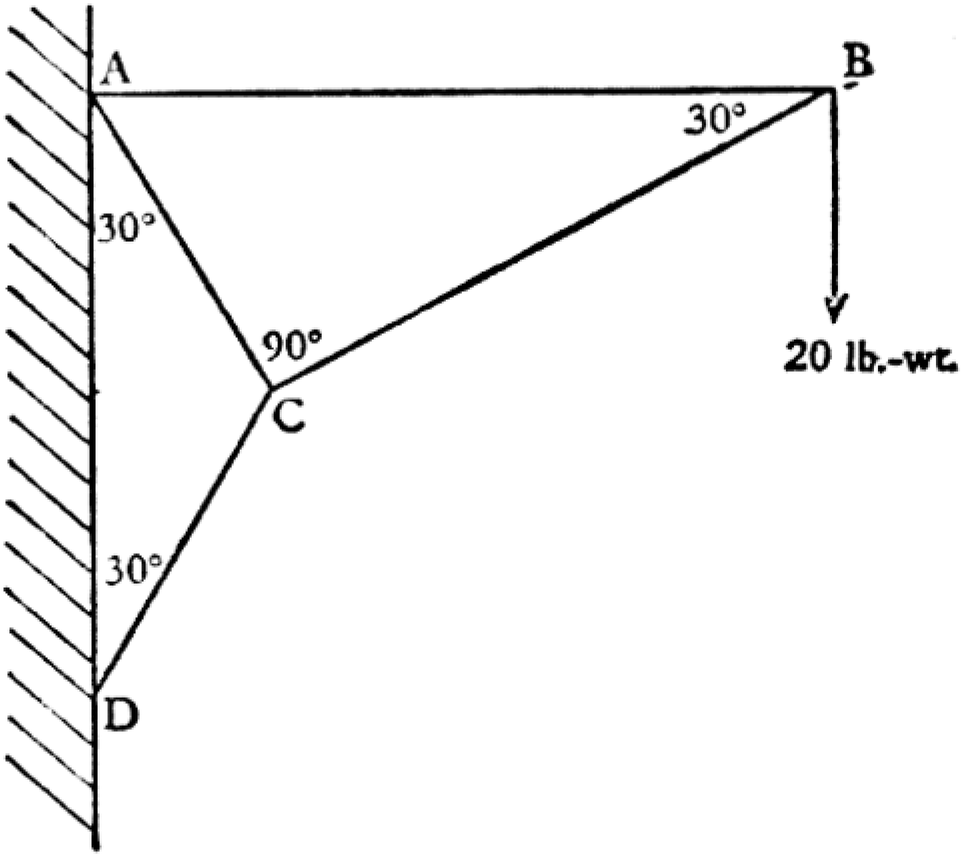
5. A pin-jointed roof-truss with the given loads is represented to scale in the figure. The truss is smoothly hinged at A and can slide horizontally at B with a coefficient of friction 0·4. Use a graphical construction to determine fully the reactions at A and at B when the truss is contracting owing to a fall in temperature.



6. A box tilted against a step is shown in the figure, the coefficient of friction between the box and the step being 0·25. The sides of the box have lengths 42 in. and 30 in., while the angle $\theta = 30°$. Find graphically the magnitude and direction of the least force applied at B which will cause the box to turn about A without slipping, if W is equal to 200 lb.-wt.



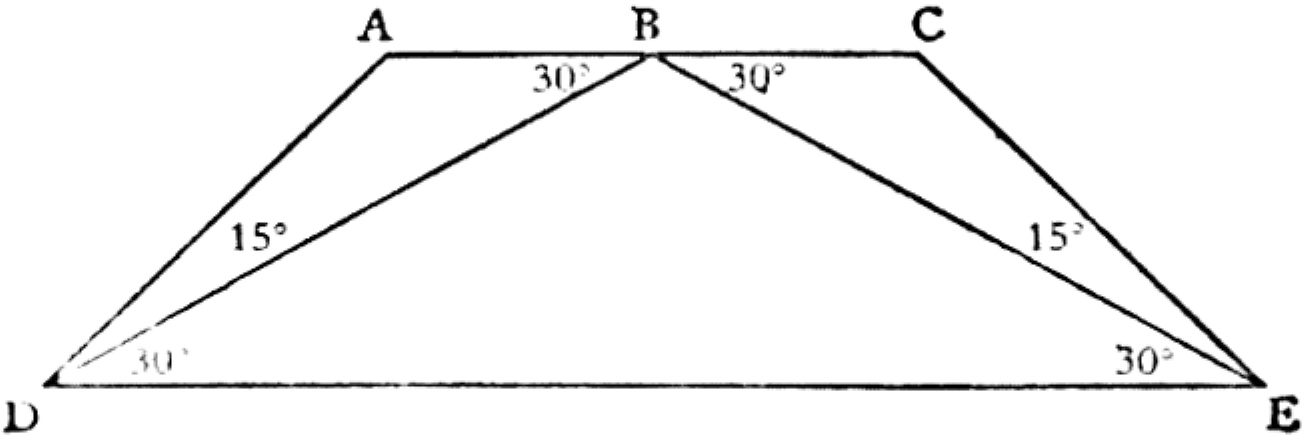
7. Four freely jointed rods, AB, BC, CA, CD, are freely jointed at A and D to fixed supports and loaded at B, as



shown. Find the stresses in the rods and the reactions at A and D.

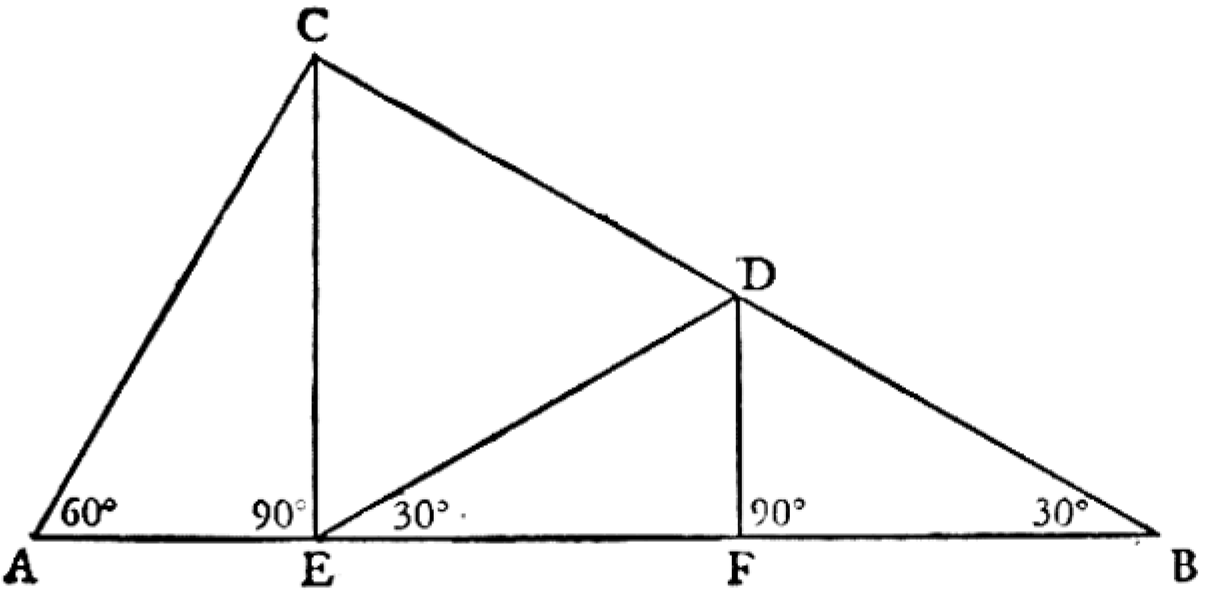
8. A smoothly jointed framework, ABCD, is placed with BC, AD horizontal, BC being vertically above AD and equal to half its length. Strengthening bars BE, CE extend from B, C to the mid-point E of AD. The triangles ABE, BEC, ECD are isosceles right-angled triangles. The frame is supported at A and D, and masses of 30 and 50 cwt. are placed at B and C respectively. Find the supporting forces, and the stresses in the bars.

9. The framework shown, made of light rods smoothly jointed together, is supported at its outer corners D, E,

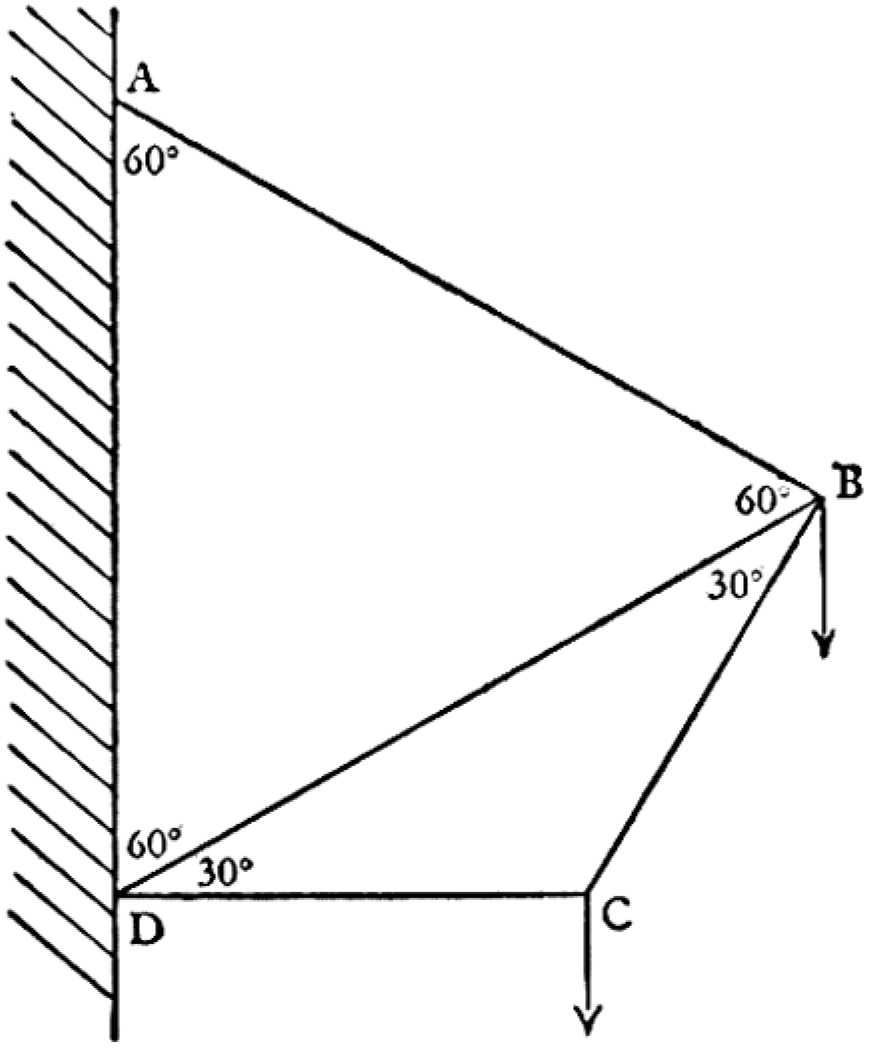


and carries loads of 1, 4, 1 tons-wt. at A, B, C respectively. Find the stress in each rod.

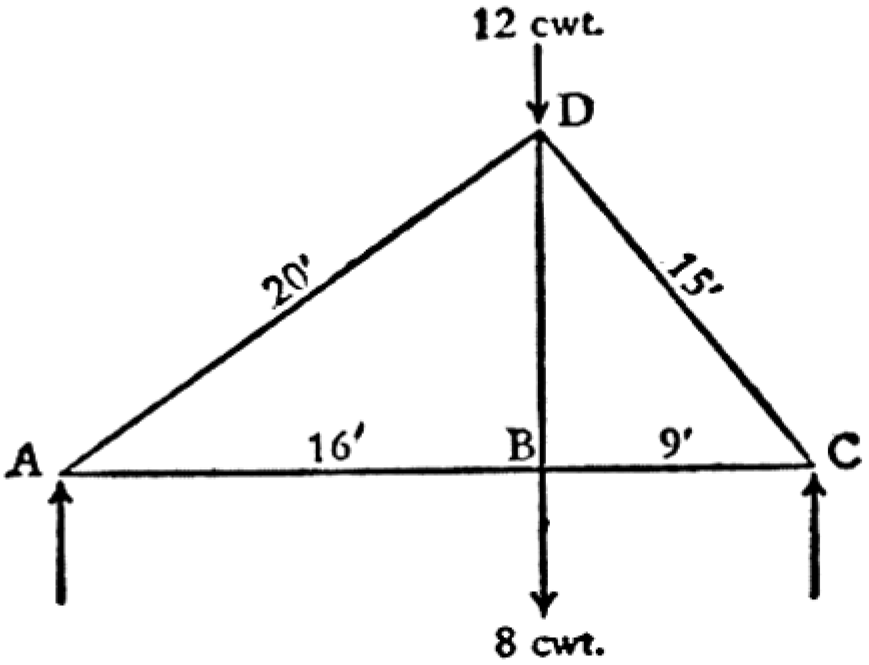
10. The freely jointed framework shown is supported at A and B and carries loads of 20 and 40 cwt. at C and D respectively. Find the stress in each member.



11. A small wall-crane, formed of jointed rods, is smoothly hinged to the wall at A and D. Loads of 10 cwt. each are fastened at B and C (see figure). Find, from a stress diagram, the stresses in the rods and the reaction at D.

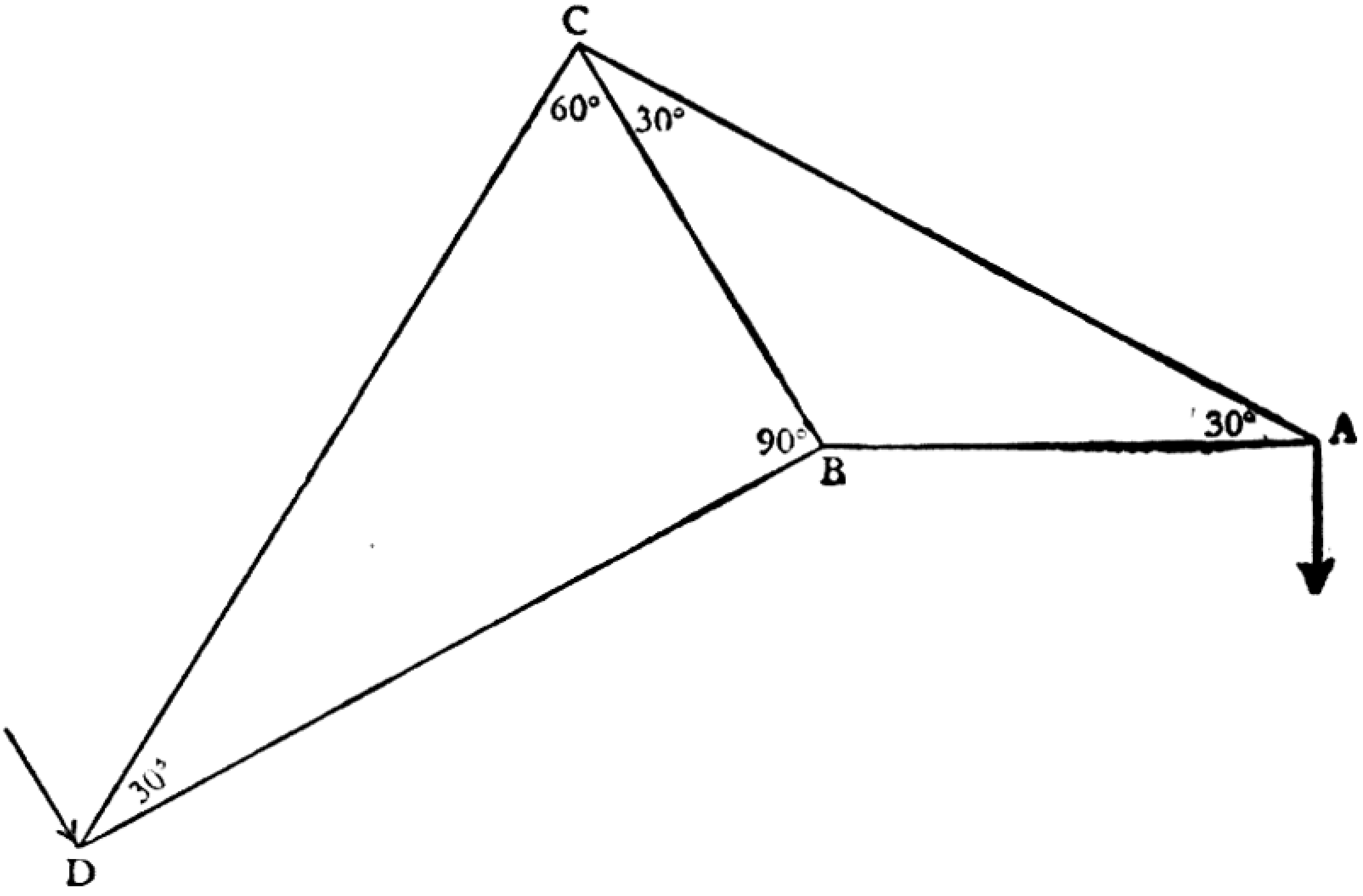


12. By means of a stress diagram, determine the stresses in the five light rods forming the framework loaded and



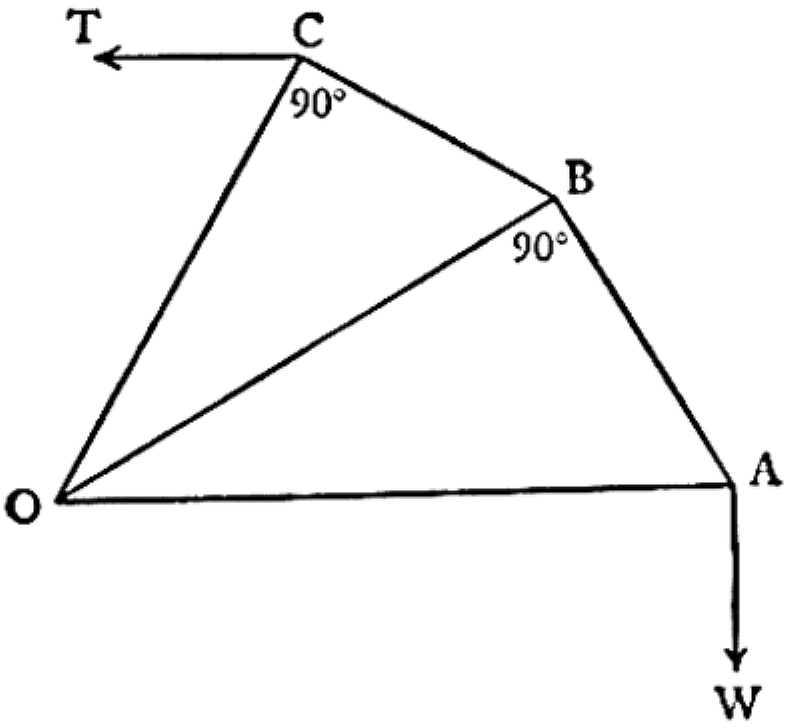
supported as in the figure, with AB and BC horizontal and BD vertical.

13. The figure represents a framework of light rods smoothly jointed at A, B, C and D; the framework is

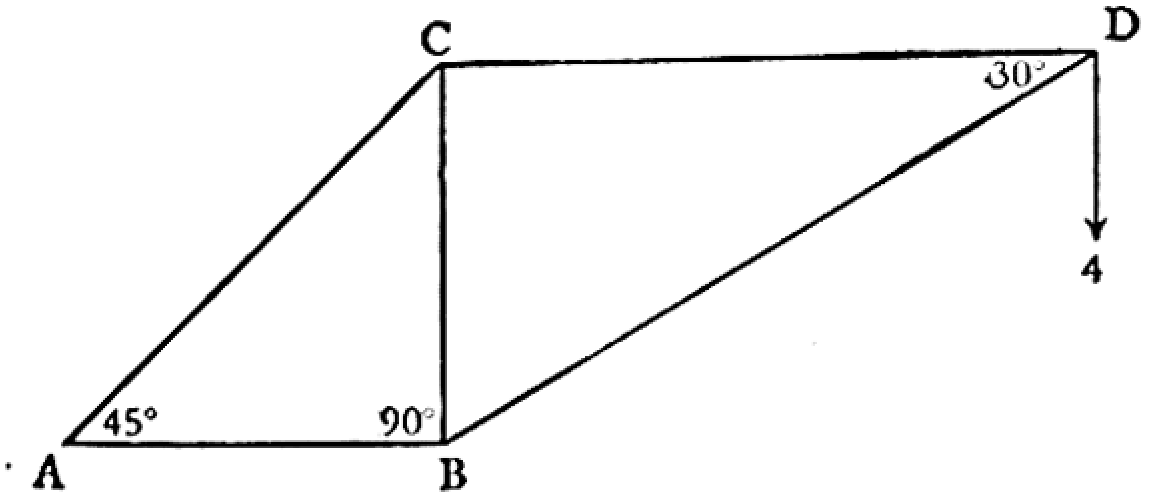


pivoted to a fixed point at C and maintained in position with BA horizontal by a reaction at D, which is perpendicular to DB. Find the stresses in the rods when a load of 2000 gm.-wt. is attached at A.

14. The given framework of light smoothly jointed rods is smoothly hinged at O, carries a load W at A, and is supported in equilibrium in a vertical plane with OA horizontal by a stay-rope attached at C. Find the reaction at O, the tension T, and the stress in each rod.

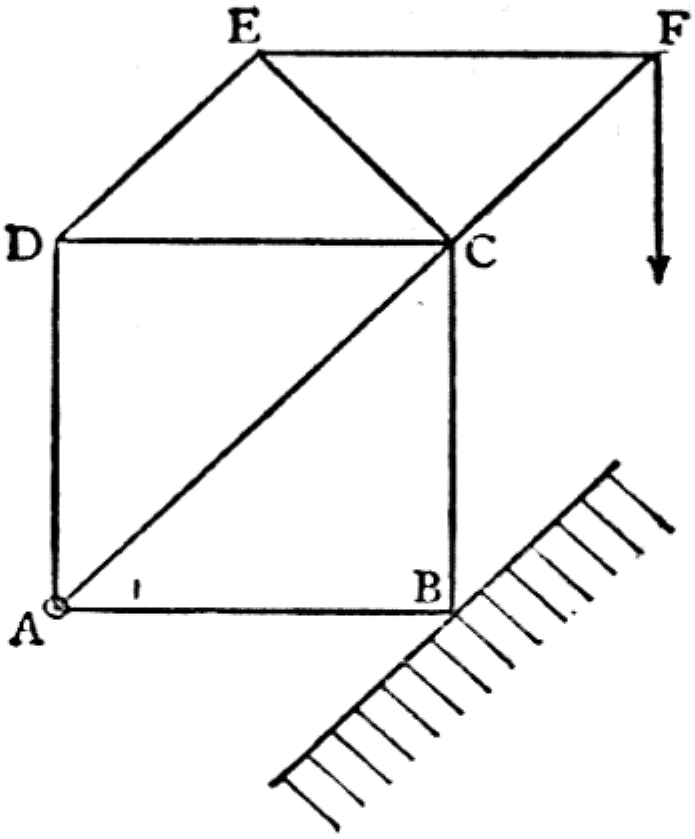


15. The framework shown is supported in a vertical plane at A, B, with AB and CD horizontal; there is a smooth pin at A, the supporting force at B is vertical, and a load of 4 tons-wt. is suspended from D. Find the stress in each rod.

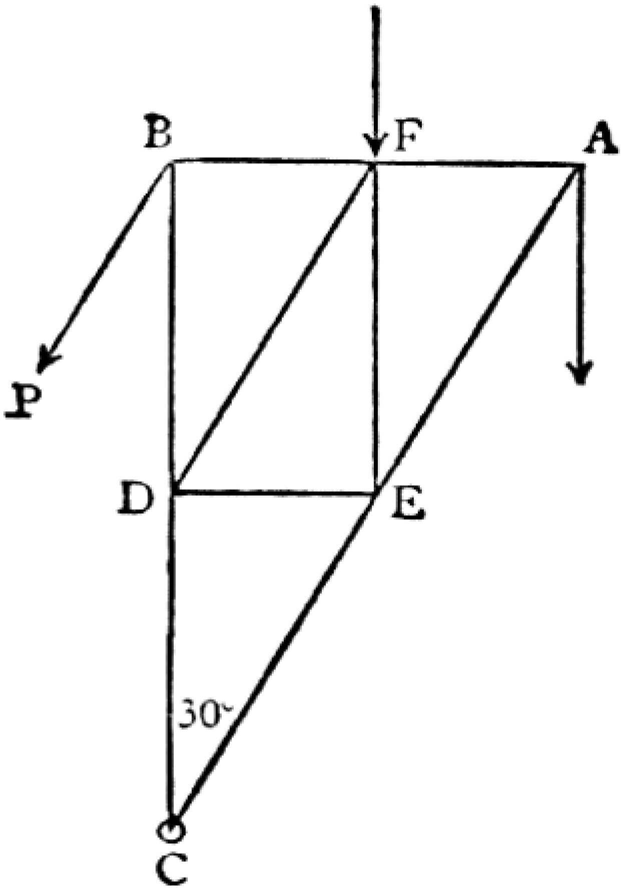


16. The figure represents a framework of nine freely jointed rods, smoothly hinged to a fixed point at A and

resting against a smooth plane at B inclined at 45° to the horizontal. All the angles are 90° or 45°. A load of W hangs from F. Find the reactions at A and B and the stresses in the rods.

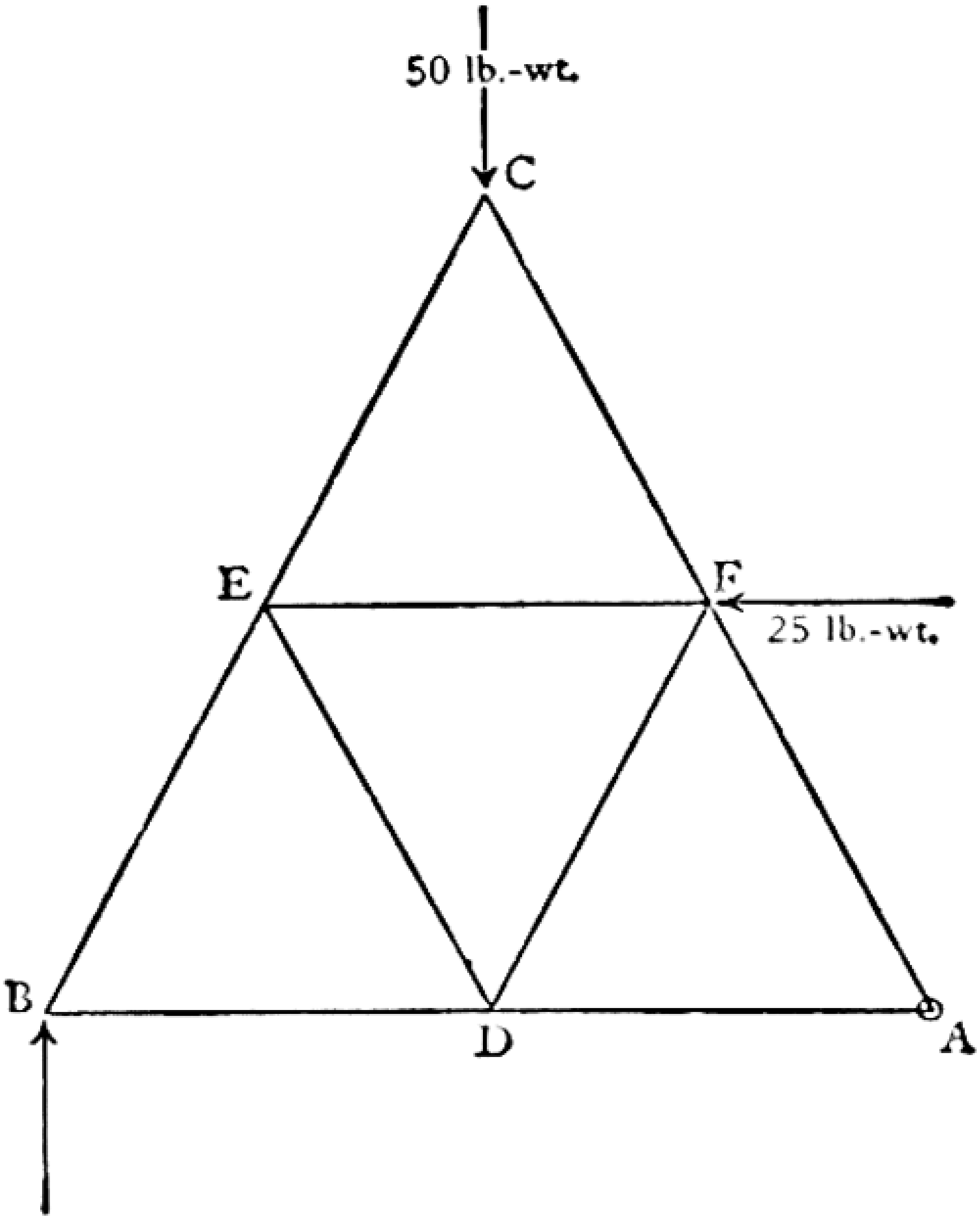


17. The figure represents a framework of smoothly jointed rods which can turn about a pivot at C. It carries loads of 3 lb.-wt. at A and F and is kept in equilibrium by a force P at B, parallel to AC and FD. AB and ED are



horizontal, BC and FE are vertical, and the angle ACB = 30°. Find the force P and the stresses in the rods forming the triangle DEF.

18. Nine freely jointed, equal, light rods, loaded as shown, are smoothly hinged at A to a fixed point and supported at B by a vertical force so that AB is horizontal. Find the force, the reaction at A and the stresses in the rods.



19. Four equal loads W are suspended by threads from points B, C, D, E of a light string ABCDEF, the ends at A and F being attached to fixed pegs on the same horizontal level. CD is horizontal and the tension in it is 3W/2. The horizontal distances between the threads attached at B and C, C and D, D and E are 3, 4, 3 in. respectively. By drawing force and funicular polygons find the inclinations of AB and BC to the horizontal, the tension in each string and the length of the portion BC.

20. A, B, C, D, E are five points in a horizontal straight line such that AB = BC = CD = DE. Loads of 5, 7, 6 lb.-wt. hang vertically under B, C, D respectively, supported by strings connecting the weights to each other and to A and E. The string joining A to the 5 lb.-wt. is inclined at 30° to the horizontal. Find the inclinations of the other strings and the tension in each.

SECTION J

1. A smooth horizontal rail is parallel to a smooth wall and one foot from it; a uniform beam, length 16 ft., rests over the rail with one end pressing against the wall; by using the principle of virtual work show that in the position of equilibrium the beam makes an angle of 60° with the horizontal.

2. ABCD is a quadrilateral of four equal sides, the angle at A being 60°. Suppose that the sides and the diagonals, AC and BD, represent six light rods connected by smooth joints, and that AC is exerting a tension T; find by virtual work the stresses in the other rods.

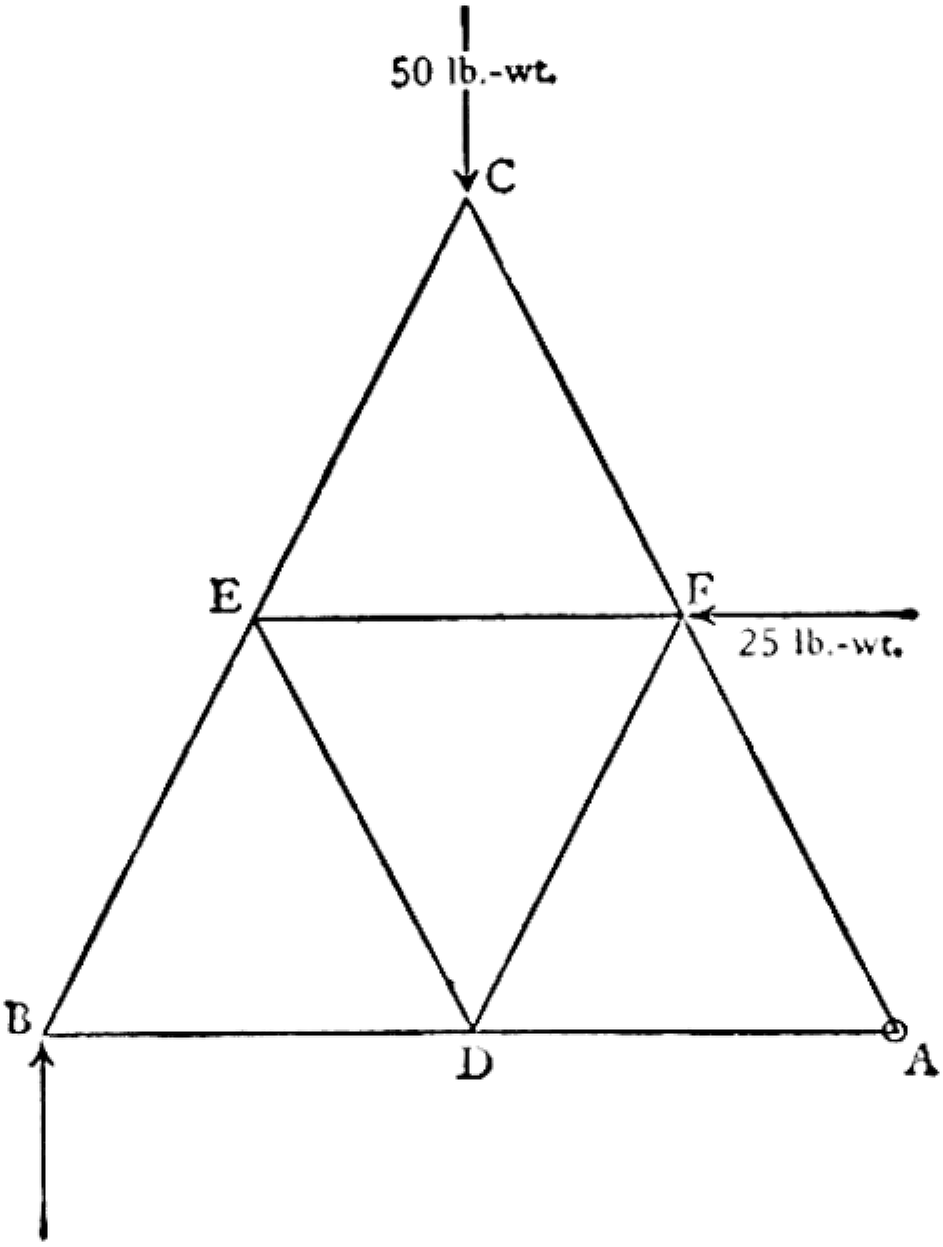
3. Four equal uniform rods, PQ, QR, RS, ST, each of length $5a$, mass $8w$, are freely jointed together at Q, R, S, and a light inextensible string, length $6a$, joins Q and S. The whole hangs freely from P and T which are distant $14a$ apart in a horizontal line. Use the principle of virtual work to find the tension in the string.

4. 20 metres of a steel wire, diameter 0·7 cm., of density 7·8 gm./c.c., are hung from the top of a tower. By how much does the wire stretch under its own weight? (Take Young's modulus as $2{\cdot}2 \times 10^{12}$ c.g.s. units.)

5. One end of a rubber cord, for which Young's modulus is 5×10^8 dynes per sq. cm., is fixed and to the other end is fastened a mass of 10 gm. which is caused to rotate in a horizontal circle at 2000 revolutions per minute. If the cross-sectional area of the cord is 5 sq. mm., and the radius

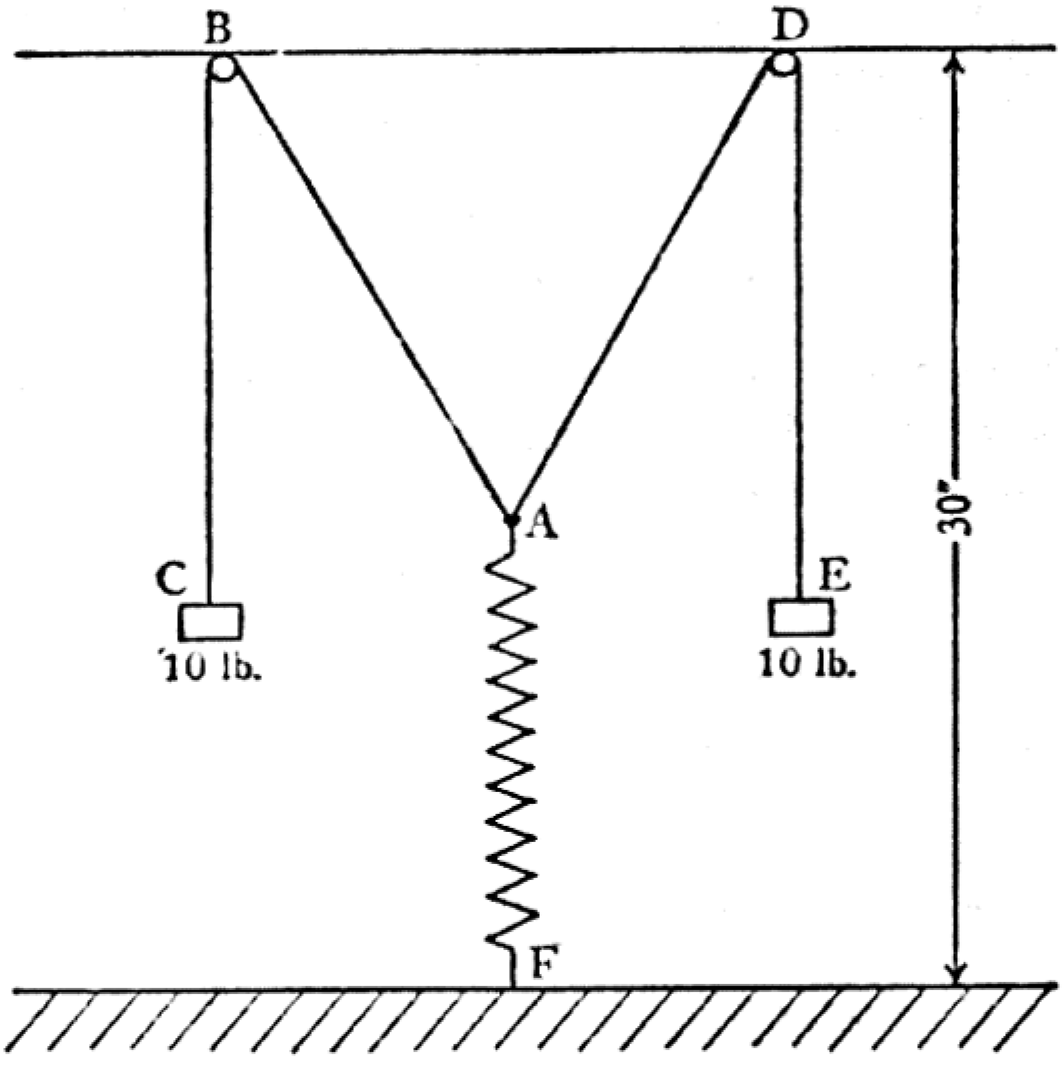
horizontal, BC and FE are vertical, and the angle ACB = 30°. Find the force P and the stresses in the rods forming the triangle DEF.

18. Nine freely jointed, equal, light rods, loaded as shown, are smoothly hinged at A to a fixed point and supported at B by a vertical force so that AB is horizontal. Find the force, the reaction at A and the stresses in the rods.



19. Four equal loads W are suspended by threads from points B, C, D, E of a light string ABCDEF, the ends at A and F being attached to fixed pegs on the same horizontal level. CD is horizontal and the tension in it is 3W/2. The horizontal distances between the threads attached at B and C, C and D, D and E are 3, 4, 3 in. respectively. By drawing force and funicular polygons find the inclinations of AB and BC to the horizontal, the tension in each string and the length of the portion BC.

9. Two strings ABC and ADE (see figure) pass over small frictionless pegs B and D in the same horizontal plane. Masses of 10 lb. are suspended at the ends C and E and the other ends are attached to the upper end A of a spiral spring AF. The lower end of the spring is fixed to the ground at F and the strength of the spring is such that a force of 4 lb.-wt. extends it 1 in. The unstretched length of the spring is 12 in., the height of the pegs above the ground is 30 in., and the vertical through F bisects the line BD. In the position of equilibrium it is found that AB = AD = 15 in. Find the distance BD.



10. If a punch, length 6 in., of material for which $E = 3 \times 10^7$ lb./sq. in. be struck with a force of 50 tons-wt. in order to form a hole of $\frac{3}{4}$ in. diameter in a block, by how much is the punch shortened in the work?

11. A steel beam is supported from a rigid ceiling by two steel wires, length 3 m., diameter 0·5 cm., whose axes are 4 m. apart. The beam is horizontal when a mass of 60 kgm. is suspended at its centre. Find the difference be-

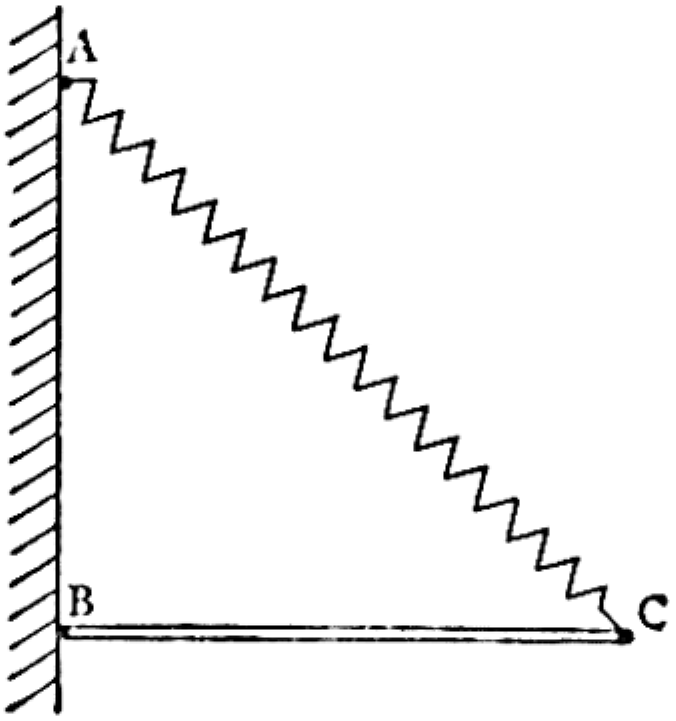
of the circle described is 12 cm., find the unstretched length of the cord.

6. Two uniform bars, AB, AC, each of mass W, length $2a$, are freely jointed at A and stand in equilibrium in a vertical plane with their ends B, C on a smooth horizontal plane and their mid-points joined by a light elastic string of natural length $2x$ and modulus λ. If 2θ is the angle between the rods, show that θ satisfies the equation

$$\frac{1}{\sin\theta}+\frac{W}{\lambda\cos\theta}=\frac{a}{x}.$$

7. A heavy particle is attached to the mid-point of a light elastic thread whose natural length is 65 cm. If the ends of the thread are attached to two pegs on the same level at distance 72 cm. apart, the particle hangs 27 cm. below the level of the pegs. Show that the depth of the particle, if the span between the pegs is increased to 96 cm., is reduced to 20 cm.

8. The light rod BC, shown in the figure, is hinged to the wall at B and supported by the spring AC, which



stretches half an inch per lb.-wt. of tension in it. AB = BC = 2 ft. If a mass of 10 lb. is hung from C, find the extra force in the spring. The length of the spring is such that BC is horizontal before loading.

$\dfrac{a(\mathrm{P}-\mathrm{Q})}{4\mathrm{P}}$ from the centre of the shelf. Assume the shelf to rest on vertical supports at its ends.

18. Prove that a uniform flexible inextensible string, hanging freely under gravity, assumes the form of the curve $s = c\tan\psi$, and show that the tension at a point at distance y from the directrix is wy, where w is the line density of the string. The distance apart of two telegraph poles is $2a$, and the telegraph wire can sustain a maximum tension equal to the weight of a length L of the wire, a/L being small. Show that the sag at the mid-point must not be less than $a^2/2\mathrm{L}$ approximately.

19. The end links of a uniform chain, length l, are free to slide on two smooth rods in a vertical plane, which make the same acute angle α with the downward vertical at their point of intersection. Show that the sag at the middle of the chain is

$$\tfrac{1}{2}l\tan\tfrac{1}{2}\alpha.$$

20. One end of a uniform chain of length l is fixed to a point A. After passing over a smooth peg B on the same level as A and at a distance $2a$ from it, the chain hangs freely and vertically. The length of the chain is so great that the sag, d, of the part of the chain between A and B is small compared with a. Show that, approximately,

$$d = \frac{a^2}{2(l-2a)}.$$

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tween the lengths of the two wires which results when the mass is moved 1·5 m. to one side of the centre. (Young's modulus for steel: $2{\cdot}2 \times 10^{12}$ c.g.s. units.)

12. Draw diagrams to represent the shearing stress and the bending moment for a light horizontal beam, length 12 ft., supported by a prop 1 ft. from one end and a prop 2 ft. from the other end, and loaded with 36 lb.-wt. at its mid-point.

13. A uniform plank, mass W, length $2a$, is supported at its ends and loaded with 2W at a distance $a/2$ from one end. Draw a graph to show the bending moment and find its greatest value.

14. Work the preceding example for the case when the load is in two parts, each W, at distances $a/2$ from each end.

15. A uniform beam, length 6 ft., mass 60 lb., is supported in a horizontal position, one support being at one end of the beam and the other at a point 1 ft. from the other end. Draw a graph to show the bending moment at any point of the beam, and find the point at which its absolute value is greatest.

16. A light beam, AB, length 6 ft., has equal loads of 20 lb.-wt. suspended from B and the mid-point C. It is supported in a horizontal position by props at A and at a point D distant d ft. from A, where $d > 4\frac{1}{2}$. Find the reactions at A and D in terms of d, and sketch the bending moment diagram for the beam. Show that the bending moment is numerically greatest at D if $d < 3\sqrt{3}$, and find the point where the bending moment vanishes when $d = 5$.

17. The right-hand half of a light bookshelf is filled with equal books of total weight P; the left-hand half is filled with equal but lighter books of total weight Q. The length of the shelf is $2a$; find the bending moment at any point and show that this moment is a maximum at a distance

before the ball returns to his hand, and show that the ball will have travelled a distance equal to

$$\frac{v^2}{g}\left\{1+\frac{2(n+1)}{n^2}\right\}.$$

6. Four stations A, B, C, D on a railway are so placed that $AB = \frac{1}{2}BC = CD = 20$ miles. A train has maximum speed of 50 miles per hour, to which it can accelerate uniformly in 10 min., and from which it can decelerate uniformly to rest in 5 min. Find the shortest time in which the train can travel from start at A to stop at D (*a*) stopping for 2 min. at B and at C, (*b*) without stopping at B or C.

7. Two bombs are dropped simultaneously from two aircraft at the same altitude. One plane is flying horizontally, the other is rising with an upward velocity of 42 miles per hour. One bomb hits the ground 2 sec. after the other. Neglecting air-resistance, calculate the altitude of the aircraft at the moment of bomb-release. Had air-resistance been considered, would the result be greater or less than that now obtained?

8. In a 1-mile cycle race, one competitor had a start of $14\frac{1}{2}$ sec. on a second competitor. The first competitor attained his maximum speed, which he retained to the end of the race, in $\frac{1}{2}$ min.; the second rider attained his, 3 ft. per second higher, maximum speed at which he then travelled steadily, with the same acceleration. If the result was a dead-heat, find the speeds of the competitors.

9. A train starting from rest is timed over successive distances of 220 yd. with the following result:

Distance in yards,	220	440	660	880	1100	1320
Time in seconds,	45	63·6	78	90	100·6	110·2

Plot a space-time graph and a velocity-time graph. Is the acceleration uniform? What is its average value?

10. Two particles move in straight lines over equal distances from rest to rest. The first particle has constant

acceleration f for the first third of the time, constant speed for the second third, and constant retardation f for the last third of the time. The second particle has constant acceleration f for the first third of the distance, constant speed for the second third, and constant retardation f for the last third of the distance. Show that the times taken by the two particles are in the ratio $3\sqrt{3} : 5$.

11. A train starts from rest with an acceleration of 3 ft. per second per second, which decreases uniformly to zero in 33 sec. Prove that the train has then acquired a velocity $33\frac{3}{4}$ miles per hour, and find the distance travelled.

12. A plane flies over a straight course along which are placed at 1-mile intervals a series of marking pylons. It was noted that the speed of the plane as it passed over successive pylons was 180, 204, 223, 238, 226, and 210 miles per hour. Deduce figures that will enable you to plot a distance-time graph for the plane, and read off the time taken over the middle 3 miles of the course.

13. The acceleration f of a particle at certain times t in its motion is given in the table below. Plot an acceleration-time curve, deduce the velocity-time curve, and so obtain the total distance travelled by the particle between times 10 sec. and 60 sec. from the start.

t	0	10	20	30	40	50	60	70 sec.
f	5·1	4·0	3·4	3·1	2·9	2·7	2·4	1·6 ft. per sec.^{2}

14. The velocities of a motor-car as indicated by its speedometer as the car passed successive telegraph poles on a straight road were 0, 4, 11, 20, 30, 22, 16, 11, 8, 5, 0 miles per hour. (The car started and came to rest beside a pole.) If the distance between the poles was 88 yd. in each case, deduce a graph relating the acceleration of the car and the time.

15. Velocities v of a suburban train at times t from the start of a journey are given in the table. Draw a v-t

curve, and from it find the distance travelled, and the value of the greatest acceleration of the train.

t	0	20	40	60	80 sec.	100	120	140	160	180	200 sec.
v	0	5·1	13·2	19·5	24·0 ft. per second.	25·3	23·0	18·5	11·6	5·1	0 ft. per sec.

16. Taking units along the time-axis to represent seconds, along the y-axis to represent feet per second, a velocity-time curve between the times $x = 0$, $x = 4$ sec. is the arc of a circle of radius 5 with its centre on the negative y-axis, the initial velocity being $+1$ ft. per second. Find the acceleration at the instant when the velocity vanishes in this interval, and prove that the distance travelled by the point during the interval and its final displacement from its initial position are respectively 2 ft. 6 in. and 1 ft. 7 in. to the nearest inch.

17. The table gives related values of s and v, the distance travelled from rest, and the velocity acquired of a body. By plotting $1/v$ and s deduce such values as will enable a graph of velocity against time to be drawn, and so find the time at which the velocity was greatest.

s	0	20	40	60	80 ft.	100	120	140	160 ft.
v	0	3·8	19·7	39·2	58·4 ft. per second.	90·0	87·3	80·0	70·0 ft. per second.

18. Corresponding values of the velocity and acceleration of an electric train are given in the table. Plot a graph relating the reciprocal of the acceleration and the velocity. Hence find the time that elapsed between the instants when the velocity was 35 ft. per second and 55 ft. per second.

v	20	30	40	50	60	70 ft. per second.
f	1·80	1·45	1·35	1·20	0·80	0·30 ft. per sec.^{2}

19. A point P moves with a given speed along a straight line; A and B are points in the line, and C is a point such that ACB is a right angle. Consider the angular velocity of P with respect to C (1) when it is greatest, (2) when P

is at A, (3) when P is at B, and show that the three angular velocities are in the same ratio as 1, $\sin^2 A$, and $\cos^2 A$.

20. A car is travelling with uniform speed and P is a point on the circumference of one of the wheels. Prove that, if θ is the angle between the direction of the velocity of P and the direction of the acceleration of P, then $\theta = 60°$ once in each revolution of the wheel, and $\theta = 120°$ once in each revolution of the wheel; and that, at each of these two instants, the direction of the velocity of P is inclined at 60° to the vertical.

SECTION L

1. An electron, mass $8{\cdot}8 \times 10^{-28}$ gm., is moving with velocity 16,000 Km. per second. If the velocity is halved in a distance of 5 cm. by the action of a constant force, find the value of the force.

2. A balloon is falling vertically to the ground with velocity 5 ft. per second and acceleration $1\frac{1}{2}$ ft. per second per second when 25 lb. of ballast are dropped from it, the remaining mass of balloon and contents being 800 lb. A quarter of a minute later a further 25 lb. of ballast is dropped out. After how much longer will the balloon again be at the same height as it was when the first ballast was jettisoned?

3. Water is flowing through a pipe of diameter 4 in. so that the rate of discharge is 5 gall. per second. If the pipe contains a right-angled bend, what is the force exerted by the water at the bend?

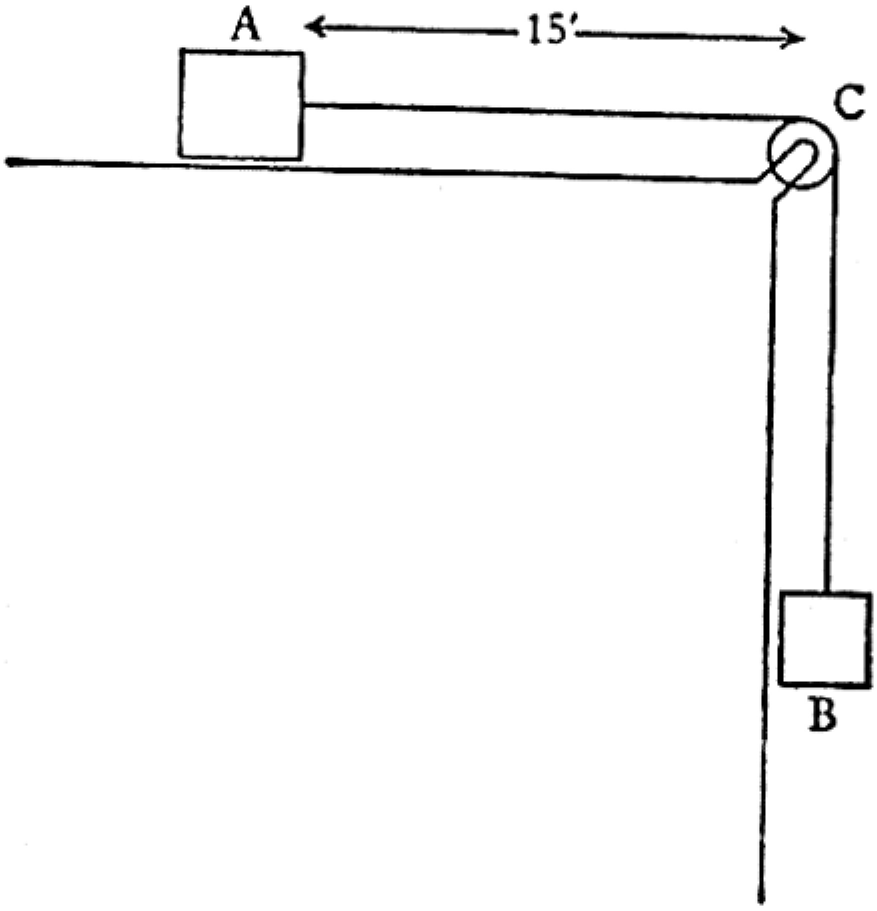
4. Masses of $\frac{1}{2}$ lb. and $2\frac{1}{2}$ lb. hang from the ends of a light string, length 8 ft., passing over a small smooth peg 12 ft. above the floor, the mid-point of the string being on the peg. The masses are released from rest and, as the smaller reaches the peg, the larger becomes detached from the string. With what velocity does it hit the floor?

5. A train, mass 120 tons, runs from rest to rest, a dis-

tance of 1 mile. The engine exerts a constant pull of 2 tons-wt. during the early part of the run, after which steam is shut off and the brakes applied simultaneously. Prove that, if the coefficient of friction between wheels and rails is $\frac{1}{7}$ and if one-quarter of the limiting friction is called into play, the time of running will be about 170 sec.

6. A train, mass 200 tons, speed 30 miles per hour, is $\frac{1}{4}$ mile from a station when steam is shut off. Track resistance totals 2 tons-wt. and maximum brake resistance is ten times this. Where should the brakes be applied if the train just stops at the station? If the brakes had been applied at the same point, but only so as to exert one-half their maximum resistance, show that the train would have passed through the station with speed very nearly 16 miles per hour.

7. A mass, A, of 3 lb. is connected to a mass, B, of 7 lb. by a fine string passing over a small pulley, C. Frictional resistances opposing the motion of the masses are $\frac{1}{2}$ lb.-wt. due to the roughness of the table, and $\frac{1}{2}$ lb.-wt. due to the pulley. From the position shown in the figure, when B is 4 ft. above the ground, the system is allowed to move. As B hits the ground the string breaks. Determine whether A will reach the pulley.



8. A certain railway line is level from A to B and on an upward gradient of 1 in 100 from B to C. A train is travelling up the gradient BC at a steady speed of 20 m.p.h. and at the instant when the rear wagon is $\frac{1}{2}$ mile from B its coupling breaks. The rear wagon continues up the incline for a short way until it comes to rest and it then rolls back under the influence of gravity. Friction causes a retarding force of 15 lb.-wt. per ton at all speeds. Find the distance from B of the point where the wagon eventually comes to rest, and sketch the velocity-time curve.

9. A uniform wedge, mass M, has identical triangular cross-sections parallel to the section ABC through its centre of mass, the angle at A being α. The face containing the side AB is rough, the others are smooth. The wedge is placed on a smooth horizontal table with the face containing AC in contact with the table, and a particle, mass m, slides down BA. If f and f' are the accelerations of the wedge relative to the table and of the particle relative to the wedge respectively, prove that

$$(M + m)f = mf' \cos\alpha.$$

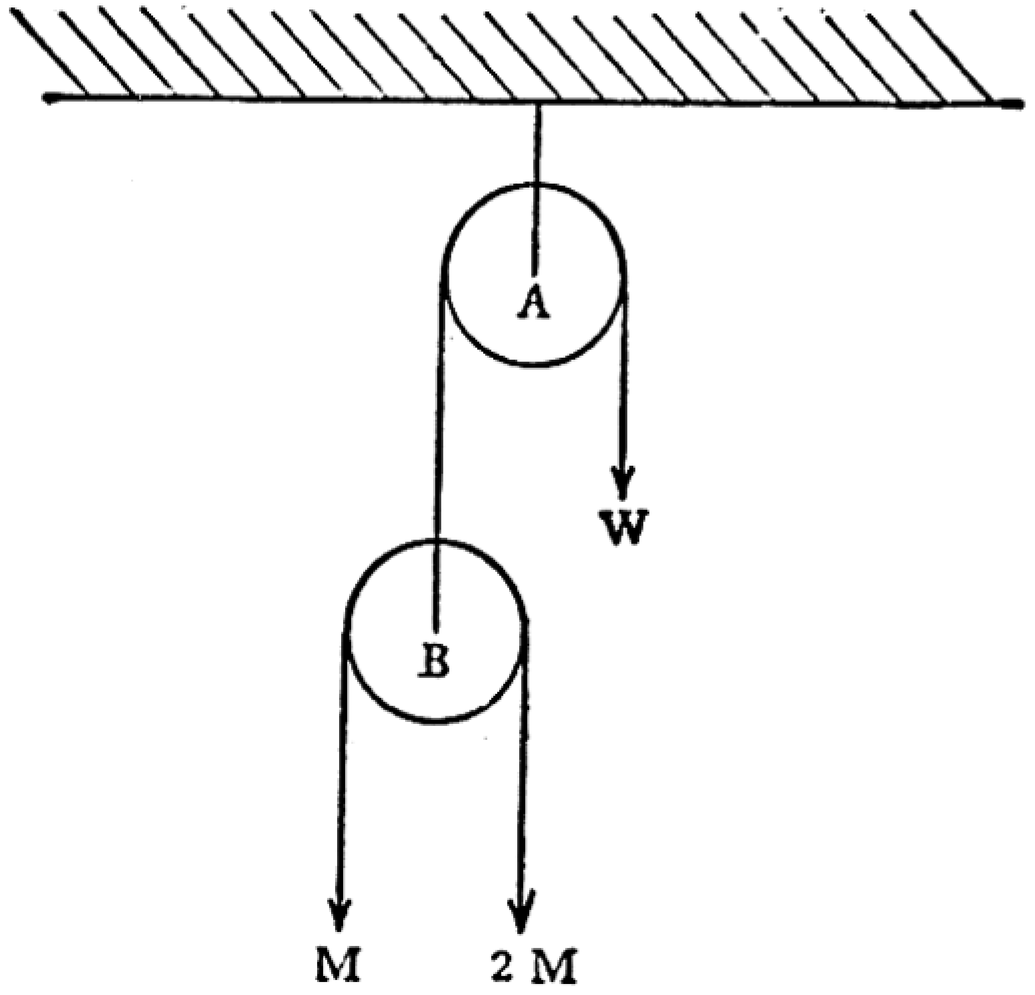
If the coefficient of sliding friction between the particle and the wedge is $\tan\alpha$, show that f and f' vanish.

10. On a mountain railway two cars move on parallel tracks, each inclined at 30° to the horizontal. The cars, masses 15 tons and 20 tons, are connected by a cable passing over a wheel at the top of the incline. Frictional resistances for each car total 20 lb.-wt. per ton; brake resistances are B lb.-wt. per ton. If the heavier car is on the point of descent and motion is prevented by applying its brakes only, show that the least possible value of B is 245. Assuming this value, in what distance would the cars be stopped, by application of both sets of brakes, if they were travelling at 8 m.p.h.? Consider separately the cases of the heavier car moving (i) upward, (ii) downward.

11. A hill road rises uniformly 19 ft. for every 181 ft. of its length. A car climbing the hill at 39 m.p.h. approaches

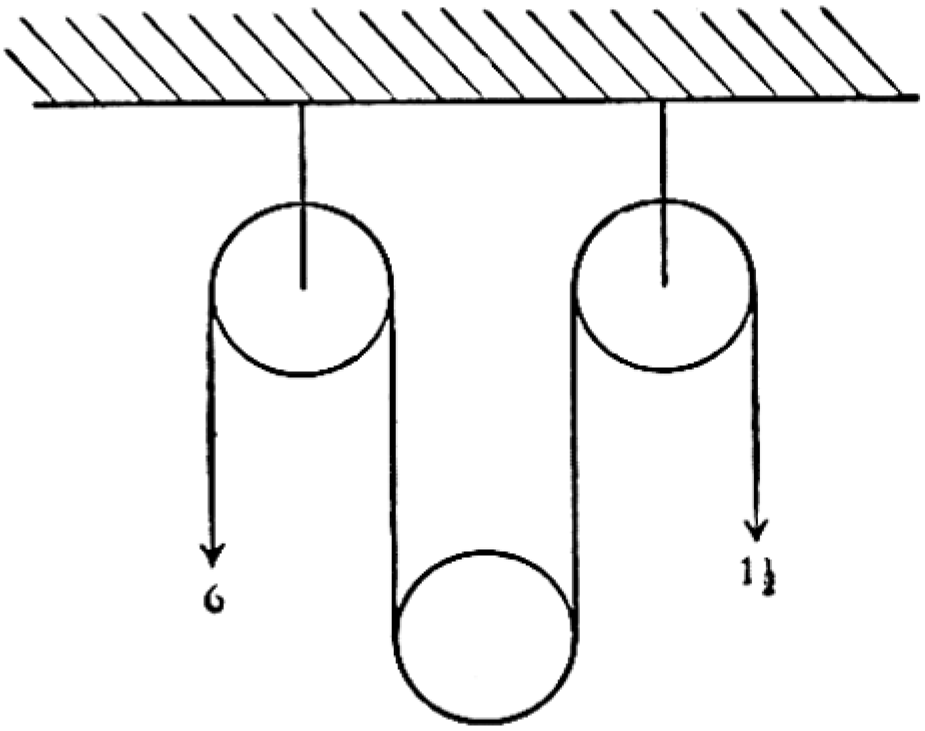
a car descending the hill at 53 m.p.h. The road being too narrow for the cars to pass, the drivers apply their brakes simultaneously, obtaining the maximum friction between the road and wheels, which is $\frac{2}{5}$ of the normal reaction between the road and wheels in each case. Show that there will be no collision if the cars were more than 141 yd. apart when the brakes were first applied.

12. The figure shows a fixed pulley A, over which passes a string carrying a mass W and a pulley B, over which passes a string carrying masses M and 2M. Neglecting friction and the inertia of the pulleys, find the acceleration with which W moves in the cases (*a*) when the mass of B may be neglected, and W = 3M, (*b*) when the mass of B is M, and W = 4M.



13. A man, who cannot exert a vertical force exceeding 100 lb.-wt., stands in a stationary lift holding a bag, mass 60 lb. The lift moves upwards with constant acceleration for a certain distance and then comes to rest under gravity, the whole distance ascended being 100 ft. Calculate the shortest time in which the ascent can be made, if the man holds the bag all the time.

14. Masses of 6 lb. and $1\frac{1}{2}$ lb. hang from the free ends of the string which passes over two fixed pulleys and under one movable pulley, as shown in the figure. The system being released from rest, the movable pulley remains at the same vertical distance below the fixed pulleys. Find the mass of the movable pulley and the acceleration of the 6 lb. mass.



15. A seconds pendulum is situated in a lift. Suppose that the lift is moving with upward acceleration f_1 ft. per second² during 1 hr. of each day, and with downward acceleration f_2 ft. per second² during $\frac{1}{2}$ hr., while for the remainder of the time it is either at rest or moving with uniform speed. Show that the number of seconds which the pendulum gains in one day is

$$\frac{225(2f_1 - f_2)}{8}.$$

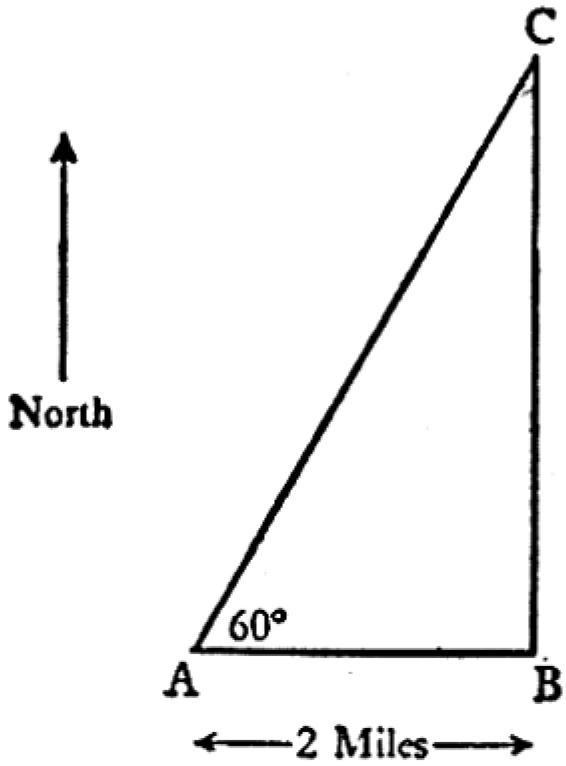
SECTION M

1. A plane flying due west in still air is travelling at 300 miles per hour. It enters a region where the wind is blowing at 42 miles per hour from the north-west. Find (*a*) the new velocity of the plane relative to the ground,

(*b*) the direction in which it must be headed so that it may again be travelling due west.

2. To a man running south-east at 10 miles per hour the wind appeared to be coming from the south. When he walked at 4 miles per hour the wind appeared to come from the south-west. Find the velocity (magnitude, and direction — assumed horizontal) of the wind.

3. A plane flies repeatedly over a triangular course ABC (shown in the figure), taking 40 sec., 1 min., and 1 min. 20 sec. to travel over the legs AB, BC, CA respectively. What vector changes in velocity occur at A, B, C if it be assumed that the speeds over each portion are uniform?



4. In a triangle ABC side AB is 6 in. long, angle ACB is 105° and angle ABC 30°. If it revolves in its own plane about the point C with a uniform speed of 2 revolutions per second, find the relative speed of A with respect to B.

5. A boy runs with uniform speed of 12 miles per hour round a circular track, while a man walks along a diameter of the circle with a speed of 4 miles per hour, both starting at the same instant from the same point. Find the velocity of the boy relative to the man (i) at the initial instant, (ii) when the boy has covered $\frac{1}{4}$ of the track, (iii) when he has covered $\frac{2}{3}$ of the track.

6. A plane which can complete a course in the form of a

square in time $4t$ at a constant speed V when there is no wind, flies over the same course with a wind of velocity kV (where $k < 1$) blowing parallel to one side. Prove that the time will be $2t\left\{\frac{2-k^2}{1-k^2}\right\}$ and find the time taken if the wind, velocity kV, had been blowing parallel to a diagonal of the square.

7. Two motor buses, breadth b, move one behind the other with velocity v along a narrow street and there is a clear space of length s between them. Show that the least velocity with which a pedestrian could go across between them is $\frac{bv}{\sqrt{s^2+b^2}}$ and find in what direction he should cross.

8. A batsman is at the wicket W and a fieldsman is in the outfield at F. The batsman strikes the ball in a direction making 30° with WF with a speed $1\frac{1}{2}$ times that with which the fieldsman can run. If the fieldsman starts at once at top speed so as to field the ball as soon as possible, determine the direction in which he must run; and show that, if in doing this he has run 20 yd., he was standing about 39 yd. from the wicket. (Assume that the ball travels along the ground with no diminution of speed.)

9. A car is driven at constant speed round a square whose sides run due east, due north, due west and due south; the apparent direction of the wind makes angles θ_1, θ_2, θ_3, θ_4 east of north. Show that, if the wind actually remains constant in strength and direction, then

$$\tan\theta_1 + \tan\theta_3 = \frac{4}{(\cot\theta_2 + \cot\theta_4)} = \frac{2(\tan\theta_1 - \tan\theta_3)}{(\cot\theta_2 - \cot\theta_4)}.$$

10. A rowing-boat which can be propelled at 3 miles per hour in still water is caught in a current moving at 4 miles per hour to the north-west. The rower wishes to reach a

point 6 miles to the north. Show that there are two possible directions in which to head the boat, and find the times taken on such courses before the objective is reached.

11. A, B, C are three towns which are at the vertices of an equilateral triangle of side 120 miles. An airman wishes to fly from A to B and then from B to C. His speed in still air is 150 miles per hour, and when the wind is blowing at right angles to AB, his course on leaving A is in the direction bisecting the angle BAC internally. Find the speed of the wind and the time taken for the journeys from A to B and from B to C.

12. A convoy steams due east at 12 knots; a destroyer leaves the convoy at noon and proceeds due N.E. at 30 knots. After 1 hr. the destroyer is instructed to rejoin the convoy in the shortest possible time at 30 knots. Find the direction in which the destroyer must steam and the time when it regains the convoy.

13. Two ships leave a port, one at 12 noon on a course due north and with speed 8 knots, the other at 1 p.m. with speed 10 knots. The course of the second is north-west for two hours and thence north-east. When were the ships closest together, and what was then the separating distance in sea-miles, after 3 p.m.?

14. The wind blows steadily from the east at w miles per hour and the cruising speed of an aeroplane in still air is v miles per hour. Show that if the pilot wishes to fly in a direction α E. of N., he must steer on a bearing $(\alpha + \beta)$ E. of N., where $\sin\beta = (w/v)\cos\alpha$, and to return in the opposite direction α W. of S., he must steer on a bearing $(\alpha - \beta)$ W. of S. If he has fuel for t hr. at his cruising air-speed of v miles per hour, and must return to his starting-point without refuelling, show that he must start the return journey before t_1 hr. have elapsed, where

$$t_1 = \tfrac{1}{2}(1 + \tan\alpha \,.\, \tan\beta)t.$$

15. Small rings A and B start from O at the same time and slide down smooth straight wires OP and OQ respectively, which are in the same vertical plane. OP makes an

angle $\alpha (< 45°)$ and OQ an angle $90° - \alpha$ with the horizontal, OP and OQ being on the same side of the vertical through O. Show that the acceleration of B relative to A is vertical and of magnitude $g(\cos^2 \alpha - \sin^2 \alpha)$.

SECTION N

1. Determine the direction of projection (for a given velocity) which produces the greatest range on a horizontal plane. A cricketer throws the ball 100 yd. Prove that, neglecting air resistance, the time taken is about $4\frac{1}{3}$ sec., if he throws in the direction determined above. Prove also that the necessary velocity of projection is about 67 miles per hour.

2. A projectile takes t_1 seconds to reach a certain point in its trajectory, and t_2 sec. to travel from that point to the horizontal level of projection. Prove that the height of the point is $\frac{1}{2}g \cdot t_1 \cdot t_2$ ft.

3. A boy A throws a cricket-ball to another boy B, the ball having a velocity whose horizontal and vertical components are 40 and 52 ft. per second respectively when it leaves A's hand; B catches the ball with his hand at the same level as A's. How far from A is B standing? If B throws the ball back to A, the horizontal and vertical components of the ball's velocity when it leaves B's hand being 52 and 40 ft. per second respectively, prove that A should catch the ball without moving his hand.

4. A particle being projected with given velocity and given angle of elevation, find the equation of its path, taking the point of projection as origin and the horizontal line through this point as x-axis. If two points on the upward part of the path are (10, 9), (20, 16), find the highest point reached and the horizontal range.

5. The greatest height reached by a projectile above the horizontal through the point of projection is 18 ft. and its horizontal range is 200 ft. Find its velocity of projection.

6. A shell is fired with velocity 72 ft. per second from the top of a 200-ft. cliff to hit an object 150 ft. away from the foot of the cliff in a line perpendicular to the vertical through the position of the gun. Find the possible angles of projection and show that the corresponding times of flight of the shell are very nearly $6\frac{1}{4}$ and $2\frac{1}{2}$ sec.

7. A bomber pilot, flying parallel to a straight road and vertically above it, sees a tank approaching from the opposite direction. The pilot goes into a dive at 400 ft. per second at $\tan^{-1}\frac{4}{3}$ to the horizontal, and releases a bomb from a height of 12,800 feet at the instant when the nose of the bomber is pointing directly towards the tank. Five seconds after his first bomb has exploded he releases another which secures a direct hit. Assuming that the speed and angle of the dive are maintained constant, prove that the speed of the tank was approximately 23 miles per hour.

8. A 60-ft. flagpole is struck halfway up by a stone fired with velocity $8\sqrt{15 + 10\sqrt{3}}$ ft. per second from a point $10\sqrt{3}$ ft. from the base of the pole. Find the direction of projection. Show that the necessary velocity of projection, if, with the same angle and initial point, the stone had just passed over the top of the pole, would be slightly over 124 ft. per second.

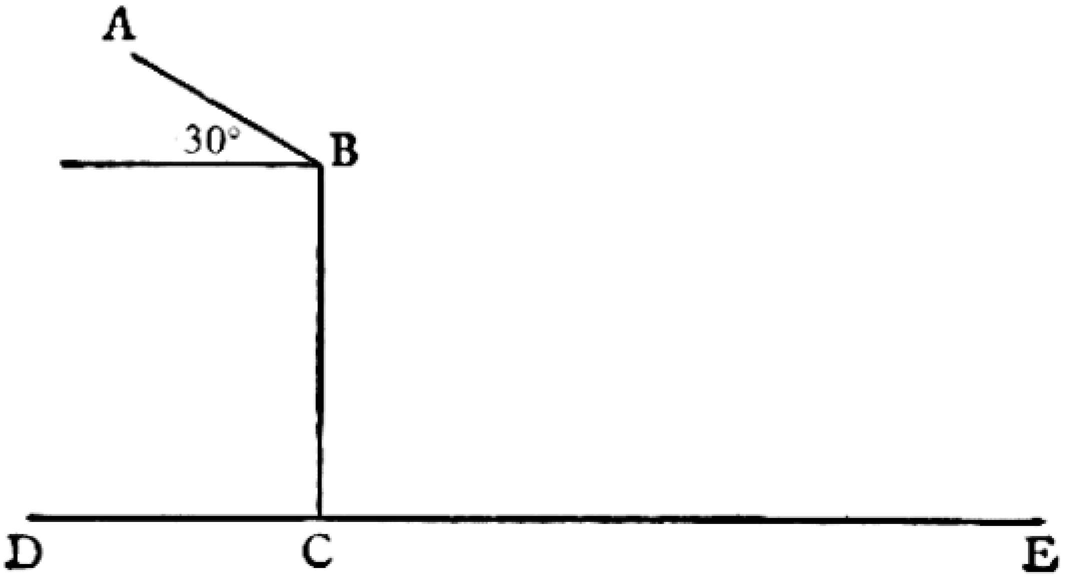
9. To " convert " a try at Rugby, the football is kicked over a bar 10 ft. high. Assuming it directed towards the middle of the goal in a direction perpendicular to the bar from a point 50 ft. away, find the least possible velocity of projection and the corresponding angle.

10. If the horizontal component of the initial velocity of a projectile is three times its vertical component, and if its horizontal range is 240 ft., find all you can about its motion.

11. A balloon, rising vertically from the ground with constant acceleration f, was struck by a shell fired at the instant when the balloon was released. If the shell was fired from a gun on the ground at distance a from the point

of release of the balloon, show that the least possible initial speed of the shell was $\sqrt{a(f+g)}$ and find the angle of elevation of the gun for this speed of projection.

12. A laboratory experiment consists in allowing a small marble to slide from rest along an inclined plane AB, and travel through space to hit the horizontal through DC at E (see figure). Neglecting all resistances, if AB is inclined at 30° to the horizontal and has length 6 ft., while BC = 10 ft., calculate a value for CE. Would you expect the experimentally obtained value to be greater or less than this?



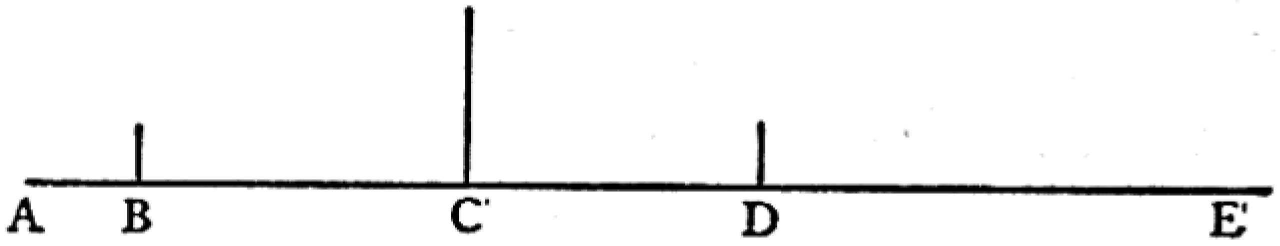
13. A ball thrown from a point 30 ft. away just clears a wall 12 ft. high. Its velocity of projection being 40 ft. per second, find the possible angles of elevation.

14. If a projectile fired from the ground level just clears an obstacle of height b at a horizontal distance a, and reaches the ground again at distance c from its starting-point, prove that the angle of projection is $\tan^{-1}\left\{\frac{bc}{a(c-a)}\right\}$. From this formula verify the following construction: If A is the starting-point, B the top of the obstacle, and C the farthest point, draw CB and produce it to cut the vertical through A at D, draw DE horizontal to cut the vertical through B at E: then AE is the direction of projection.

15. If R is the greatest horizontal range possible for a

particle projected with a given velocity, without any restriction, prove that when the vertical height is restricted not to exceed $h(<\frac{1}{4}R)$ the greatest range attainable is $2\sqrt{2h(R-2h)}$.

16. A gunshot fired from A just clears obstacles at B, C, D, hitting the horizontal plane through ABCD at E (see figure). The heights of the obstacles at B and D are

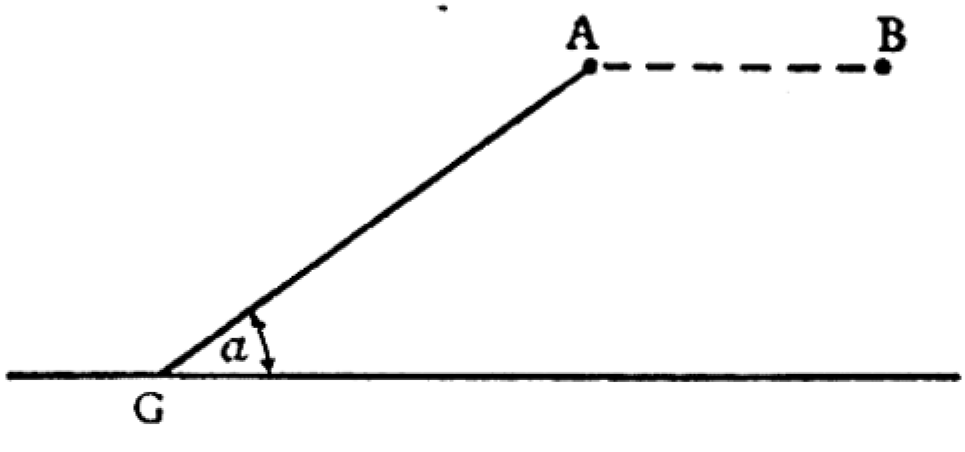


each 30 ft., that at C is 90 ft. high, and the distances AB, BC are 60, 180 ft. respectively. Calculate the velocity of projection of the shot and the position of D.

17. A body projected at an angle of elevation α is at height h above the horizontal through the point of projection twice during its flight. The horizontal distances of such points from the point of projection being s_1 and s_2, prove that

$$\tan\alpha = \frac{h(s_1 + s_2)}{s_1 s_2}.$$

18. A plane is flying horizontally with constant velocity v ft. per second. A shot, fired from a gun G (see figure)



when the plane is at A, hits the plane when it is at B, t sec. later. Prove that $v = \frac{1}{2}g \cdot t \cdot \cot\alpha$.

19. From a tower of height h a body is projected with a velocity due to a height nh at an angle of projection equal

to $\tan^{-1}\left(\frac{n}{n+1}\right)^{\frac{1}{2}}$. Find the range on the horizontal plane and show that with the given velocity it cannot be exceeded.

20. If the greatest height attained by a rifle bullet is $1/n$th the range on a horizontal plane through the point of projection, show that, unless $n = 4$, it is possible to increase the range by altering the angle of projection.

21. An aircraft, cruising speed in still air 210 m.p.h., flies horizontally directly against the wind at a height of 10,000 ft. The pilot releases a bomb and, if there were no wind, it would have struck its objective. He finds it strikes $\frac{1}{12}$ of a mile short. What is the wind speed, air resistance being neglected? On the return journey with the wind, he makes the correct allowance for the wind speed, but the bomb oversteps the objective by the same amount. Show that the bomb was released about 1·35 sec. too late.

22. A shot fired with velocity V at elevation θ strikes a point P on the horizontal plane through the point of projection. Find the range of P from the point of firing. If P is receding from the gun with velocity v, show that elevation must be changed to ϕ where $\sin 2\phi = \sin 2\theta + 2v \sin\phi/\mathrm{V}$, assuming that the range of P at the instant of firing is the same as in the first case.

23. Three particles are projected simultaneously, in the same vertical plane, from the same spot. The first is projected horizontally with velocity v_1, the second at an angle θ to the horizontal with velocity v_2, and the third vertically with velocity v_3. Prove that the three particles are always in one straight line if

$$\frac{\cos\theta}{v_1} + \frac{\sin\theta}{v_3} = \frac{1}{v_2}.$$

24. A train, length 130 ft., is travelling at 30 miles an hour along a straight level track which is crossed by a bridge. When the front of the train is 55 ft. from the bridge a stone is thrown towards the train from the parapet of the bridge, which is 25 ft. above the carriage roofs. If

the horizontal and upward vertical components of the initial velocity of the stone have the same magnitude, show that the stone hits the train if this magnitude is less than 30 ft. per second.

25. A particle is projected from the foot of a plane inclined at 30° to the horizontal. It travels in the vertical plane containing a line of greatest slope of the inclined plane. What angle of elevation will cause the particle to hit the inclined plane as far as possible from the point of projection? Compare the range up the plane with that of a similar particle projected with equal speed up the line of greatest slope, friction being neglected.

26. A particle is projected in the vertical plane containing a line of greatest slope of a plane inclined at 45° to the horizontal. If the angle of elevation of projection is $\tan^{-1} 2$, prove that the particle is travelling horizontally at the instant when it hits the plane.

27. Two particles are projected from a point on a plane inclined at 30° to the horizontal, one up the plane, one down. In each case projection is in the vertical plane through a line of greatest slope of the inclined plane, and the direction of projection makes an angle of 30° with the vertical through the point of projection. (*a*) If the initial velocity of each particle is 50 ft. per second, find their ranges. (*b*) With what velocity must the up-going particle start if its range is equal to that of the down-going in (*a*)?

28. A body is projected with velocity u in a direction inclined at α to the horizontal so that it strikes at right-angles a plane, through the point of projection, inclined at β to the horizontal. Prove that $2\tan(\alpha - \beta) = \cot\beta$, and that the range on the plane is

$$\frac{2u^2 \sin\beta}{g(1 + 3\sin^2\beta)}.$$

29. Shells can be projected at any elevation with a fixed speed of projection from a coastal fort at a height H above sea-level. Show that the danger-zone for hostile

vessels is bounded by a circle of radius $2\{h(\mathrm{H} + h)\}^{\frac{1}{2}}$, with its centre vertically below the fort, h being the greatest height above the fort to which shells from the fort can be projected.

30. A shell exploding in the ground showers earth with varying velocities of projection over a circle of radius 162 ft. Show that earth falls on a hut 81 ft. away during an interval of time equal to $9\sqrt{2}/4$ sec.

SECTION P

1. A body, starting from rest, slides down a plane inclined at 30° to the horizontal. If the coefficient of friction is $\frac{1}{4}$, show that the ratio of the kinetic energy at any instant to the loss in potential energy is $(4 - \sqrt{3})/4$.

2. A hemisphere of lead, mass 840 lb., radius 10 in., resting with its curved surface on a foundry floor is turned so that its plane surface is on the floor, as little work as possible being done in the process. Find this minimum quantity.

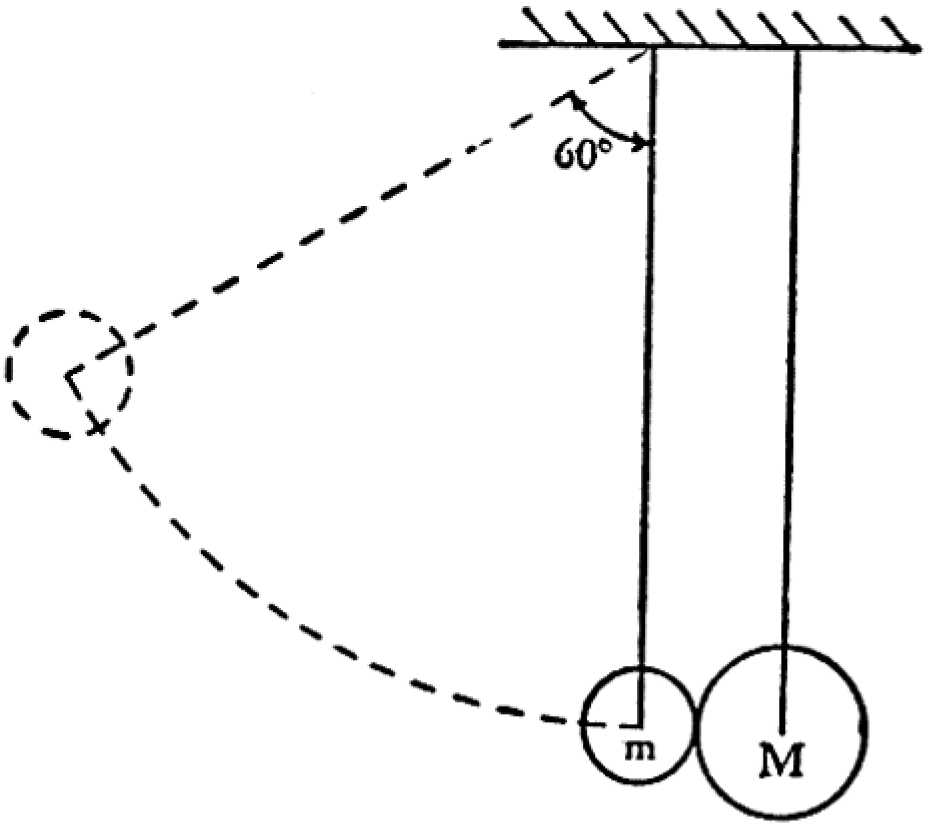
3. If the effective mean pressure obtained in the single cylinder of a steam engine is $\frac{9}{10}$ that shown by the theoretical indicator diagram, find the brake horse-power available in the engine when it is supplied with steam at a pressure of 160 lb.-wt. per sq. in., cut-off at one-third stroke, back pressure 26 lb.-wt. per sq. in. The total length of the stroke is 3 ft., the cylinder diameter 20 in. and the mechanical efficiency 85 per cent; number of strokes per minute $= 100$.

4. A fire-engine pumps water through a pipe, diameter 6 in., at such a rate that the jet, if sent vertically upwards, would reach a height of 25 ft. If the engine is 75 per cent efficient, at what horse-power is it working?

5. A delivery van, average loaded mass 25 cwt., travels repeatedly over a route involving 1 mile of level ground, $\frac{1}{2}$ mile up a slope of average gradient 1 in 100, and a corre-

sponding $\frac{1}{2}$ mile downhill. Assuming a uniform speed of 15 m.p.h., constant track resistances totalling 40 lb.-wt. per ton, and neglecting other factors, find which would be the more economical basis of propulsion — petrol or electricity. The price of petrol is 1*s.* 10½*d.* per gallon, rate of consumption 45 miles per gallon. Electricity costs 2*d.* per kilowatt-hour, and the necessary battery-charging may be considered as 80 per cent efficient. What would be the chief factor neglected?

6. Two elastic particles, masses M and *m*, are suspended by light strings as shown in the figure. The mass *m* is raised until its string makes an angle of 60° with the vertical and is then released. Assuming that the bodies coalesce, show that, if M is equal to 3*m*, after the first impact the string of the heavier mass will rotate through an angle $\cos^{-1}\frac{7}{8}$.



7. To do the cutting work in a small screw-cutting lathe it is found that 0·47 horse-power is required, and that the frictional losses in the gearing, bearings, etc., absorb another 0·21 horse-power. How many foot-pounds of work per minute is the driving belt giving to the lathe? The countershaft is driven by an electric motor, and the countershaft and belt absorb 0·17 horse-power. How

many watts must the motor be given in order to keep the lathe running, if its efficiency is 80 per cent?

8. Assuming that the engine of a car is working at a constant rate and that resistances due to friction, etc., are constant, find the horse-power of the engine if the car, mass 35 cwt., can travel on a level road at 50 m.p.h. and up an incline of 1 in 8 at 15 m.p.h.

9. A cyclist riding on a level road at 12 miles an hour is working at 0·1 horse-power. Assuming that the resistance apart from gravity varies as the square of the speed, find the rate at which he must work to ascend a gradient of 1 in 20 on the same road at 6 miles an hour, the combined mass of rider and bicycle being 200 lb.

10. An engine developing 600 horse-power and weighing, with the train, 250 tons-wt., is travelling on the level against frictional resistance of 4000 lb.-wt. Find the acceleration of the train when its speed is 25 m.p.h., and show that the maximum possible speed of the train is less than $56\frac{1}{4}$ m.p.h.

11. An engine draws a train up an incline of 1 in 100 at a uniform speed of 30 m.p.h. Together they weigh 200 tons-wt. If frictional resistances total 7 lb.-wt. per ton, find the horse-power at which the engine is working. At the top of the incline the line becomes level. Find the initial acceleration of the train.

12. A locomotive draws a train of mass 240 tons up an incline of 1 in 120 at a uniform speed of 36 m.p.h., the total resistance being $10\frac{1}{2}$ lb.-wt. per ton. Find the effective pull of the engine and its horse-power. If the engine exerted an additional 28 horse-power on the train, what immediate acceleration would be produced?

13. A car which develops 24 horse-power and has mass 1 ton can climb a slope of 1 in 10 at 30 m.p.h. when working at its full rate. Find the frictional resistance to motion at this speed. If the frictional resistance is proportional to the speed, estimate the greatest speed at which the car could travel on the level.

14. The frictional resistance to a train, mass 240 tons, travelling up an incline of angle a, where $\sin a = \frac{1}{120}$, is $240(3{\cdot}2 + 0{\cdot}003v^2)$ lb.-wt., where v ft. per second is the speed. If the maximum speed up the slope is 30 m.p.h., show that the horse-power (assumed constant) at which the engine is working is about 531.

15. A train travelling up an incline of 1 in 80 with the engine working at its maximum horse-power has a maximum speed of 45 miles per hour. Later on, with the engine still working at maximum horse-power, it attains a speed of 60 miles per hour on the level. It then starts going downhill. What must be the gradient (vertical rise/distance along incline) of this hill, if, with the steam shut off and without brakes, there is a retardation of $\frac{11}{30}$ ft. per second^2? The resistance due to friction is assumed the same throughout.

16. A train, mass 200 tons, is accelerated uniformly from rest to its maximum speed of 60 miles per hour in 6 min.; it continues with this speed for 15 min., and is then brought to rest by shutting off steam and applying a brake force of 65 lb.-wt. per ton. If the motion is subject to a constant resistance R lb.-wt. per ton, find R if the engine works at 384 horse-power when the speed is 60 miles per hour, and find the total work done against this resistance during the journey.

17. The accelerating force acting on a body, moving in a straight line, is related to the time in such a manner that the force-time curve is a semicircle, the diameter representing time, and lines drawn perpendicular to the diameter to the curve representing force. The scales are such that the total time is 60 sec. and the maximum force is 20 lb.-wt. The body has a mass of 500 lb. and starts from rest. What will be the velocity after 15, 30, 45 and 60 sec.? What is the acceleration 15 and 45 sec. from the start?

18. A train accelerates at a constant rate f from rest at one station until it is necessary to decelerate at a constant rate f' to come to rest at the next station at a distance a

away. Show that the time taken for the journey is T, where

$$\mathrm{T} = \left(\frac{2a(f + f')}{ff'}\right)^{\frac{1}{2}}.$$

If $a = 5$ miles, $\mathrm{T} = 12$ min., the total mass of the train is 220 tons, and $f' = 2f$, find the horse-power developed by the engine just before decelerating, given that the resistances then amount to $\frac{5}{6}$ of the forward force exerted by the engine.

19. When the speed of a car on level ground is 40 m.p.h., the engine is stopped and the car, with no brakes applied, runs for 27 sec. before stopping. If the mass of the car is 15 cwt., show that a little over 16 horse-power is required to take it up a hill of 1 in 45 at a steady speed of 40 m.p.h.

20. A leather belt is moving horizontally at 12 ft. per second. A slab of iron weighing 10 lb.-wt. is gently laid on the belt, the coefficient of friction between iron and leather being 0·3. Calculate (i) the time taken for the slab to acquire the velocity of the belt, (ii) the distance through which the belt has then moved relative to the slab, (iii) the total amount of extra energy supplied by the prime-mover driving the belt. What happens to this energy?

21. An engine, drawing a train of mass 400 tons, is working at 1200 horse-power when the train is travelling at full speed on the level. The resistance to motion is $0{\cdot}009\mathrm{V}^2$ lb.-wt. per ton, where V is the speed in m.p.h. What is the full speed of the train on the level? What is the acceleration of the train when ascending an incline of 1 in 112 at the instant when the velocity is 30 m.p.h. if the law for the resistance remains the same and the engine is still working at the rate of 1200 horse-power?

22. A cork, $2\frac{1}{2}$ in. long, is drawn slowly from the neck of a bottle which is also $2\frac{1}{2}$ in. long. Find the work done in extracting it, assuming that the force exerted is proportional to the length of cork in the neck, the pull at starting (when the end of the cork was flush with the top of the neck) being 30 lb.-wt.

23. The accelerations of a tramcar at different distances from the starting-point are given in the table. Plot an acceleration-distance curve, and from it estimate the velocities at 50 ft. and 100 ft. from the starting-point. If the car has mass $8\frac{1}{2}$ tons and the track and wind resistances total 395 lb.-wt., find the horse-power required at the wheels at the instant the car is 40 ft. from the starting-point.

Acceleration, (ft./sec.^{2})	3·46	2·31	1·54	1·10	0·77	0·44	0·35	0·33.
Distance (ft.),	0	10	20	30	40	60	80	100.

24. The propulsive force of a motor-car, mass 2400 lb., varies uniformly with the distance travelled from 500 lb.-wt. at the starting-point to 200 lb.-wt. when the car has moved 600 ft. on a level road; and there is a constant resistance of 50 lb.-wt. If the car starts from rest, plot a velocity-space curve for the first 600 ft., and determine the horse-power being expended when the car has travelled 300 ft. from the starting-point. What is the initial acceleration of the car in miles per hour per second?

25. A body, mass 16 lb., is thrown with horizontal velocity 6 ft. per second on to a horizontal belt travelling in a horizontal direction with a constant velocity of 8 ft. per second. These two velocities are mutually perpendicular, and the vertical velocity of the alighting body is negligible. From the first point of contact to the place where slipping ceases, the distance travelled by the body across the belt in the direction perpendicular to that of the belt's movement is 2 ft. On the assumption that the frictional force is constant while slipping takes place, find the value of the coefficient of friction between the body and the belt, and calculate the power the belt is supplying while slipping occurs.

SECTION Q

1. A sphere dropped vertically from the top of a 240-ft. tower hits the horizontal plane below and rebounds. It hits again and rebounds again. At the instant that it hits a third time the sound of the second impact is heard at the top of the tower. Velocity of sound being 1100 ft. per second, find the coefficient of restitution between sphere and plane, correct to two places of decimals.

2. Two equal imperfectly elastic spheres, elasticity e, impinge directly and one stops dead. Prove that they were moving in opposite directions with velocities in the ratio $(1 + e) : (1 - e)$.

3. Two particles, masses m_1 and m_2, moving in the same straight line towards one another with a relative velocity of approach u, collide and separate again along the line with a relative velocity v. If e is the coefficient of restitution, prove that the kinetic energy lost in impact is

$$\frac{m_1 m_2(1 - e^2)u^2}{2(m_1 + m_2)}.$$

Find the magnitude P of the impulse on either particle due to the impact, and show that the kinetic energy lost can be expressed as $\frac{1}{2}\mathrm{P}(u - v)$.

4. Three particles A, B, C of equal masses lie on a smooth table in a straight line. A is projected towards B with speed u. If the coefficient of restitution for all impacts is $\frac{1}{4}$, show that there will be two impacts between A and B and that the final speeds of the particles are in the ratios 147 : 165 : 200.

5. A railway truck is moving with a velocity of 15 miles per hour. A man suddenly swings himself on to the truck from where he stands motionless on the ground. If the man's mass is 160 lb. and the mass of the truck is 2 tons, show that the truck loses a velocity of $\frac{22}{29}$ ft. per second. What difference, if any, will it make to the velocity of the truck when the man drops off?

6. Spheres of masses 5 lb. and 8 lb. moving in opposite directions along the same path, each with velocity 10 ft. per second, collide. Find the speed of each sphere after the impact in terms of the coefficient of restitution e; and show that for the spheres to be moving in opposite directions after impact e must be at least 0·3.

7. In the direct impact of two equal elastic spheres moving in the same line, the velocities before impact are 14 and -4 units while those after impact are 2 and 8 units, all these velocities being in the same direction if positive; find the coefficient of restitution, and show that $\frac{36}{53}$ of the total energy is lost in the impact. If the spheres are unequal and the velocities which are 14 and -4 units before impact are 4 and 10 units after impact, find the ratio of the masses of the spheres and show that the coefficient of restitution is the same as before.

8. If an impinging sphere strikes an equal sphere at rest obliquely, and the coefficient of restitution is $\frac{1}{2}$, show that the maximum angle through which the impinging sphere can be deflected is $\tan^{-1}(\frac{3}{4})$.

9. A particle, mass m, moving with velocity V on a smooth horizontal table, impinges on a smooth plane inclined at an angle a to the horizontal, the line of intersection of the plane with the table being perpendicular to the direction of motion of the particle. The coefficient of restitution between plane and particle is e. Prove that after the impact the particle travels freely in space for a time, and that it lands on the table or the plane according as $e >$ or $< \cot^2 a$. Prove also that the impulse at the impact is $m\mathrm{V}(1 + e)\sin a$.

10. A railway truck, mass 10 tons, moving with a velocity of 12 ft. per second on a horizontal track, overtakes and collides with another truck, mass 10 tons, moving with a velocity of 8 ft. per second. The spring buffers are such that when the two trucks in contact are moved together through a distance x ft. the reaction between them is $9x$ tons-wt. For the instant when the trucks have a common

velocity, find that velocity, the loss of kinetic energy, and the distance the buffer springs are compressed.

11. Two equal spheres, each of radius a, rest on a horizontal plane with their centres at A and B, distance d apart. A third sphere whose centre is C moves in the direction AB with velocity V and strikes the sphere, centre A, directly. The coefficient of restitution between any two of the spheres is $\frac{1}{2}$. If A strikes B at time T after the first impact, show that

$$\mathrm{T} = \frac{4d - 8a}{3\mathrm{V}},$$

and that the distance between B and C at time 2T after the first impact is

$$\frac{13d + 22a}{12}.$$

12. Two spheres of equal mass, connected by an inelastic string, length a, rest together on a horizontal plane. One sphere is projected vertically upwards with velocity $\sqrt{2gh}$, where $h > a$. Prove that the other sphere rises to a height $(h - a)/4$. What loss of energy occurs at the instant when the string becomes taut?

13. A bullet, mass 3 oz., is fired from a gun, mass 80 lb., at 1000 ft. per second. If the gun is free to move, what will be its momentum and kinetic energy as the bullet leaves? If the gun fires bullets at 300 per minute, what average force is required to keep it at rest?

14. A bullet, mass 0·5 oz., strikes at right angles, and moves on with, a small vertical target, mass 20 lb., which is suspended by a string from a point 12 in. above its centre of mass. Assuming the mass of the target to be concentrated at its centre of mass, find the speed of the bullet, if the target first comes to rest when at the level of the point of support.

15. A mass of 12 lb. moving vertically downwards with speed of 50 ft. per second strikes the top of a vertical pile, mass 120 lb., without rebounding. The pile is driven 6 in.

into shingle by the blow. Calculate the resistance to motion (assumed uniform), the time of penetration, and the fraction of the original energy lost during impact.

16. A bullet, mass $\frac{1}{16}$ lb., having a velocity of 2000 ft. per second, is fired into a fixed block of wood. The bullet passes through the block and in doing so is deflected from its original path through an angle of 30° and its velocity is reduced to 1200 ft. per second. Find the magnitude and direction of the impulse on the block of wood.

17. Two equal small spheres are suspended in contact from two fixed points by equal vertical strings. One sphere is pulled aside in the plane of the strings until the taut string makes an angle a with the vertical, and is then released. Show that after impact of the spheres, the second string will turn through an angle θ given by

$$\sin\frac{\theta}{2} = \frac{1+e}{2}\sin\frac{a}{2},$$

where e is the coefficient of resilience between the spheres.

18. A boy throws a ball horizontally with velocity 20 ft. per second towards a vertical wall 8 ft. distant. If the point of projection is 4 ft. above the ground and the coefficient of restitution between ball and wall is $\frac{2}{3}$, find where the ball first hits the ground.

19. A ball of elasticity e is thrown, from a point distance a from a vertical wall, against the wall and returns to the point of projection. Show that the velocity u, and the elevation a, of projection are connected by the equation

$$u^2 \sin 2a = ag(1 + 1/e).$$

20. A sphere falls from a height of 16 ft. and rebounds from a plane inclined to the horizontal at $\tan^{-1} 0{\cdot}8$, the coefficient of restitution being 0·64. Show that the vertical component of the velocity immediately after impact vanishes, and that the distance down the plane to the next point of impact is 42 ft., almost.

21. A body, mass 10 lb., is moving horizontally with a

velocity of 20 ft. per second at a height of 60 ft. above the ground. While passing through a point P it is struck by another body, mass $2\frac{1}{2}$ lb., moving vertically upwards with a velocity of 80 ft. per second, and the two bodies unite. Find the horizontal distance from P of the point where the combined body strikes the ground.

22. Particles, masses m_1, m_2, joined by a light elastic string, length l and modulus λ, rest on a smooth horizontal plane. They are driven apart by equal impulses I acting in opposite directions. Show that the centre of gravity remains at rest, and that the greatest extension of the string is

$$\mathrm{I}\left\{\frac{(m_1 + m_2)l}{m_1 m_2 \lambda}\right\}^{\frac{1}{2}}.$$

23. Particles, masses m, $2m$, $3m$, . . . , nm, occupy the vertices of a regular polygon ABC . . . N, of n sides, on smooth ground. Walking round the circle through the vertices, in the direction N . . . BA, at constant speed v, I administer to each particle as I pass, beginning with the particle at N, a blow of impulse $2\pi mv/n$, directed towards the centre of the polygon. Show that at the moment when I reach N again the particles arrive simultaneously at the centre, and that, if they coalesce, the whole mass comes to rest there, whether n is even or odd.

24. From a train travelling at 40 miles per hour a mail bag, mass 90 lb., is let fall into a stationary truck, mass 140 lb. Find the average force required to bring the truck to rest in a horizontal distance of 30 ft.

25. A brick, mass 8 lb., falls freely down a well. After falling 25 ft. from rest it strikes the surface of the water. The impulse at the surface of the water on the brick would completely destroy the momentum of a mass of $\frac{1}{4}$ lb. which had fallen 25 ft. The brick then continues to descend through the water with acceleration $g/2$ until it reaches the bottom of the well $55\frac{1}{2}$ ft. below the water surface. Calculate (a) the time from rest to the bottom of the well, (b) the momentum of the brick when it reaches the bottom.

SECTION R

1. A heavy particle makes complete revolutions inside a smooth, vertical, circular tube so that its speed at the topmost point is $\frac{1}{3}$ its speed at the lowest point. Prove that the ratio of the pressures of the particle on the tube when it is moving vertically and when it is at its lowest point is 5/11.

2. A particle has a light string, length a, attached to it, the other end of the string being fixed at a point O. The particle is projected from O under gravity with velocity V. Show that if the angle of projection α is such that the particle is moving horizontally when the string becomes taut, then

$$3\sin^4\alpha - 4\sin^2\alpha + 4a^2g^2/V^4 = 0.$$

Show that this will not happen for any value of α if $V^2 < \sqrt{3}ag$.

3. A heavy particle, placed at the highest point of a smooth, vertical, circular disc, is connected by an inextensible string with an equally heavy particle which is at the extremity of a horizontal radius. If motion be allowed to ensue, prove that the upper particle will leave the disc when at an angular distance from the highest point given by the equation

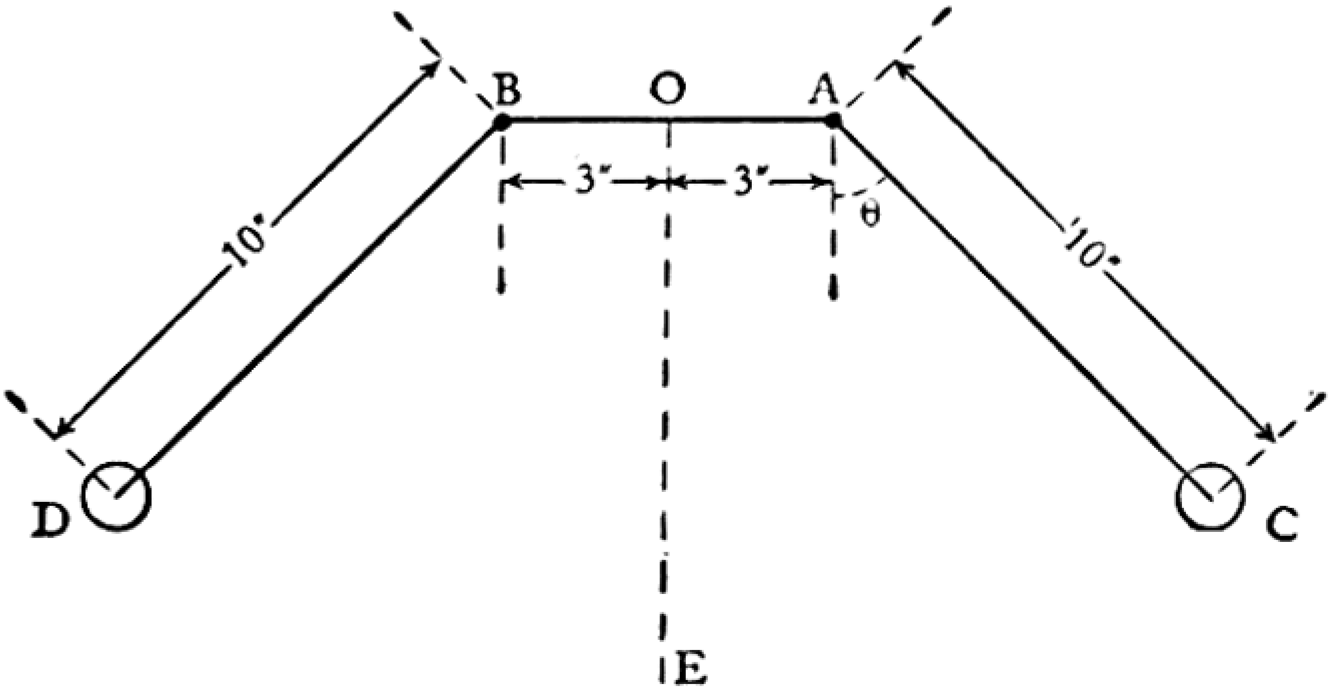
$$2\cos\theta = \theta + 1.$$

4. A particle, mass m, is attached to a fixed point by a string, length a, and is attached by another string of length a to a ring, mass n, which can slide on a smooth vertical wire passing through the fixed point. If the particle describes a horizontal circle with angular velocity ω, prove that its depth below the fixed point is given by

$$h = \frac{(m + 2n)g}{m\omega^2},$$

and find the tension in each string.

5. The horizontal arm AB (see figure) rotates about the vertical axis OE. Masses C and D are attached to the ends of rods of negligible mass freely hinged at A and B respectively so that the rods can only swing in the vertical plane containing AB. If AO = OB = 3 in., and AC = BD = 10 in., find the speed of AB in revolutions per minute when the angle θ is 60°.



6. A particle, attached by a light inelastic string, length a, to a fixed point, is describing a horizontal circle, radius r, with uniform angular velocity. The particle is suddenly reduced to rest and then let go. If, in the subsequent motion, its speed when the string becomes vertical is equal to half its speed in the original motion, prove that $49r^2 = 48a^2$.

7. A curve on a railway line is banked so that the lateral thrust on the inner rail due to a truck moving at 24 miles per hour is equal to the thrust on the outer rail when the truck is moving with a speed of 32 miles per hour. Prove that there will be no lateral thrust when the truck is moving with a speed of $20\sqrt{2}$ miles per hour.

8. A train is travelling at 30 miles per hour on a curve the radius of which is 400 yd. If the distance between the rails is 5 ft., how much must the outer rail be raised above the inner for there to be no lateral thrust on the rails?

9. A bend of a race track is banked at 45° so that if the

track were smooth there would be no tendency to sideslip at 120 miles per hour. Assuming the coefficient of friction between a car and the track to be $\frac{1}{3}$, estimate the limits of speed between which there would be no sideslip.

10. The string of a simple pendulum, of which the bob has mass m, is held horizontal and taut, and the bob is released. If the breaking tension of the string is $3mg/2$, show that the bob flies through the point at distance $7a/2$ below the point of suspension, where a is the length of the string.

11. A particle is moving with simple harmonic motion about a centre O. When it is at distance d from O its velocity is V, and when at distance $2d$ from O its velocity is V/2. Find the period and amplitude of the motion.

12. In a single vibration in simple harmonic motion displacements from the centre equal to 1, 4, and 3 ft. are observed to occur at times 0, 1, 2 sec. respectively. Determine the amplitude and the period of the motion.

13. If the displacement, velocity and acceleration at a particular instant of a particle describing simple harmonic motion are respectively 3 in., 3 in. per second, and 3 in. per second per second, find the greatest velocity of the particle and the period of the motion.

14. A particle performs simple harmonic motion on a vertical straight line AB, the time taken from A to B being equal to the time taken by another particle to fall freely under gravity from A to B. Show that the velocities of the two particles when passing through the mid-point of AB are in the ratio $\pi : 2\sqrt{2}$.

15. A light spiral spring hangs vertically from the ceiling and carries a mass of 10 lb. attached to its lower end. The stiffness of the spring is such that a force of 1 lb.-wt. extends it 1 inch. Calculate the period of oscillation of the spring. If the mass is pulled down a distance of 5 in. from its mean position and released, find its velocity $\frac{1}{4}$ sec. later.

16. O is the point of suspension of a simple pendulum,

length 6 ft. The point A is vertically below O and OA = $4\frac{1}{2}$ ft. A horizontal rail perpendicular to the plane of motion of the pendulum passes through A, so that during one half of each swing of the bob the length of the pendulum is effectively reduced to $1\frac{1}{2}$ ft. Show that the period of the pendulum is $\frac{3}{4}$ of its value T when the rail is removed. Find the depth below O of the point where the rail should be placed in order to reduce the period to 2T/3.

17. A mass of 10 lb. rests on one end of a plank, the other end of which is fixed to a wall. The free end of the plank is given a simple harmonic motion of period $\frac{3}{8}$ sec. If the mass is to keep contact with the plank, what is the greatest possible amplitude of the motion? Draw a graph to show the variations in pressure between mass and plank during a complete oscillation, and state the value of the greatest pressure.

18. A casting, mass 10 lb., the shape of which is an equilateral triangle of uniform thickness with equal circular bosses at each corner, is bolted to the face-plate of a lathe, the axis of rotation of which coincides with the axis of one of the bosses. The distance between the centres of the bosses is 6 in. and the speed of rotation is 25 revolutions per minute. What is the centrifugal force on the casting and where on the face-plate should a mass of 4 lb. be fixed in order to balance this force?

19. A mass m is attached to one end of a light inextensible string, which hangs vertically. The other end of the string is moved vertically with simple harmonic motion of amplitude a and frequency n. If the string never becomes slack, prove that $n^2 < \dfrac{g}{4\pi^2 a}$ and that, with this condition, the difference between the greatest and least tensions of the string is $8\pi^2 n^2 ma$.

20. A particle, mass m, is fixed to the mid-point of a string, length a, which is stretched to a tension T. Show that, if the particle is slightly disturbed in a direction perpendicular to the string, it will describe simple harmonic

motion of period $\pi\sqrt{ma/T}$. When the particle is at rest it is acted upon by a small impulse P perpendicular to the string. Find the amplitude of the subsequent vibration.

21. The time of low-tide in a certain channel is 8.08 a.m., and that of the following high-tide 2.23 p.m., the corresponding depths of water being 7 feet and 10 ft. 8 in. Assuming the depth to vary in simple harmonic motion, find its value at noon on the given day.

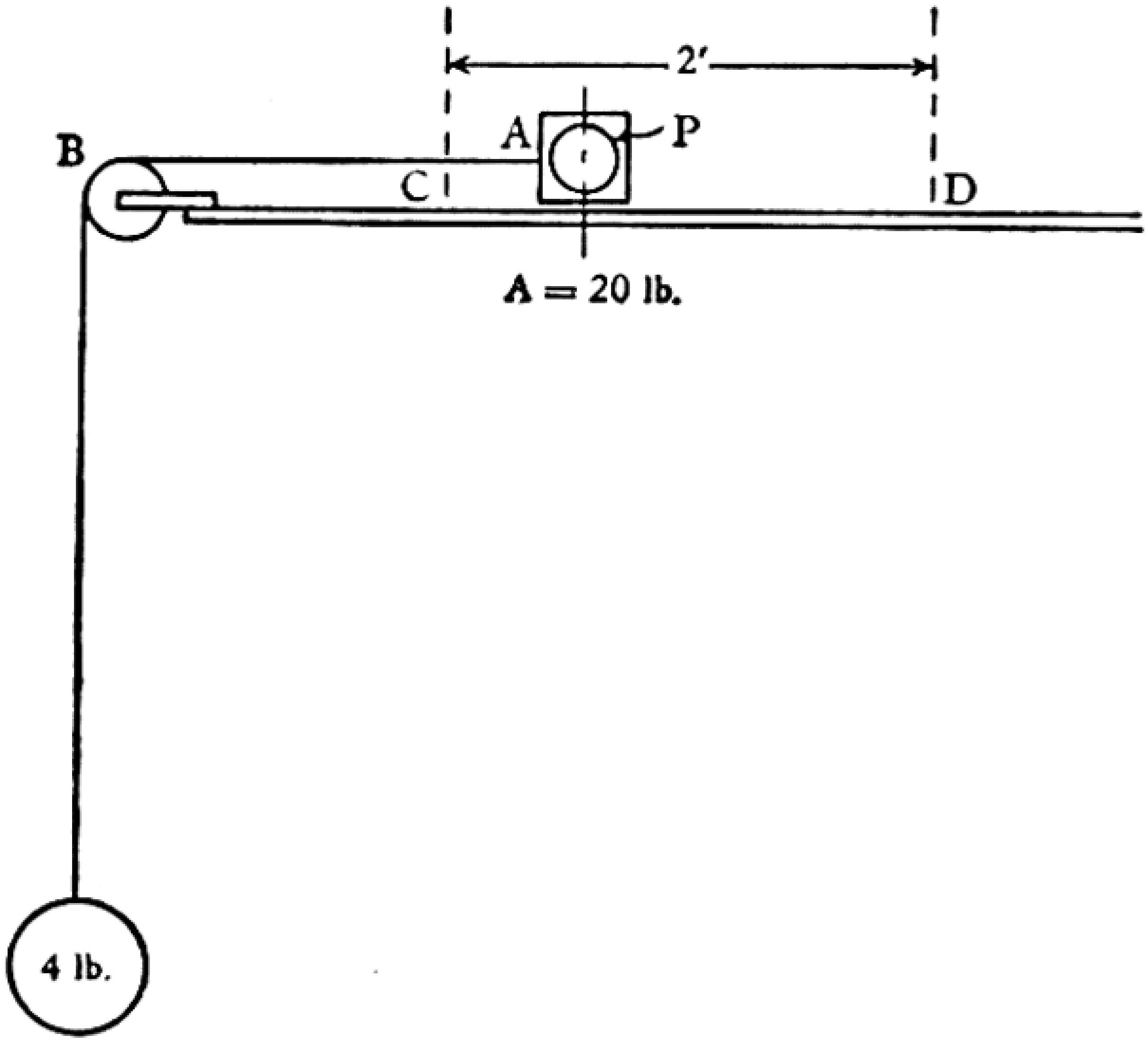
22. An engine has a crank 6 in. long and the line of stroke passes through the axis of the crankshaft. If the speed of the crank is uniform and the connecting rod very long compared with the crank, show that the motion of the piston is very nearly simple harmonic. Taking the speed of the crank as 240 revolutions per minute, find the acceleration of the piston at the ends of the stroke and the velocity of the piston at one-quarter and at one-half stroke.

23. If n equal masses m are threaded at equal intervals on a light endless inextensible string of length a, and the system rotates steadily with angular velocity ω on a smooth horizontal table, find the tension in the string. By letting $m = M/n$ and n tend to infinity, show that the tension in a heavy uniform circular flexible ring of circumference a and mass M, rotating steadily with angular velocity ω on a smooth horizontal table, is $Ma\omega^2/4\pi^2$.

24. Two exactly similar light elastic strings each have one end attached to a heavy particle, and one string is also fastened to a fixed point A and the other to a fixed point B, vertically below A. The particle rests in equilibrium at a distance b below the middle-point of AB, and between A and B, with both strings stretched. Show that, if the particle receives a vertical displacement which does not cause either string to go slack, it will perform simple harmonic oscillations of period $2\pi\sqrt{b/g}$ about its position of equilibrium.

25. A block A, mass 20 lb., is controlled through a pin P by means of suitable gearing (not shown in the figure) so that it moves with simple harmonic motion along the

horizontal surface CD. A mass of 4 lb. is attached to the block A by a light string passing over a smooth pulley B. The period of a complete oscillation from C to D and back to C is 2 sec. and the length of the path CD is 2 ft. When



the centre of the pin P is 8 in. from the end of its path C and moving away from C, the pin breaks and immediately frees A from any constraint by the harmonic gearing. If the coefficient of friction between the mass A and the surface CD is 0·25, find the position of A when it comes to rest.

SECTION 8

1. A small wheel has an axle, diameter 1 in., resting on a pair of parallel rails inclined at $\sin^{-1}\frac{1}{20}$ to the horizontal. The radius of gyration of wheel and axle about the centre line of the axle being $1\frac{1}{2}$ in., if it starts from rest find how long it will take to roll 2 ft. down the slope.

2. An armature is keyed up to a shaft, diameter 4 in., which rests transversely on two parallel steel rails inclined

at $\sin^{-1}\frac{1}{12}$ to the horizontal. The shaft rolls, without slipping and without rolling friction, a distance of 6 ft. along the rails, from rest, in 1 min. What is the radius of gyration of the armature and its shaft about the longitudinal axis?

3. A solid steel flywheel, diameter 10 in., thickness 2 in., is fixed to a horizontal steel axle, diameter 2 in., length 10 in., running in frictionless bearings. One end of a string is fixed to the axle, and a mass of 30 lb. is hung from the free end of the string. Twenty turns of the string are then wound on to the axle and the system is released. Show that the angular velocity of the wheel when the twenty turns are just unwound is almost 70 radians per second. (The mass and diameter of the string are to be neglected: the density of steel is 0·28 lb. per cubic inch.)

4. Prove that the kinetic energy of a body of mass M rotating about a fixed axis with angular velocity ω is $\frac{1}{2}Mk^2\omega^2$, where k is the radius of gyration about the axis. A flywheel, mass 3 cwt., radius of gyration 18 in., is making 180 revolutions per minute about its axis. Determine in foot-pound units the frictional couple which will bring it to rest in 50 revolutions.

5. A wheel, mass $\frac{1}{2}$ ton, external radius 3 ft., radius of gyration $2\frac{1}{2}$ ft., is rotating at 150 revolutions per minute. A friction brake applied to the outside rim of the wheel exerts a normal force of 250 lb.-wt., the coefficient of friction between brake and rim being 0·5. Find the time taken for the wheel to come to rest and the number of revolutions it makes in that time.

6. A cord, length 20 ft., of which the mass may be neglected, is wound around the axle of a flywheel. It is pulled steadily with a force of 30 lb.-wt. until all the string has uncoiled, when the wheel is moving at 240 revolutions per minute. Find its moment of inertia. If a constant frictional force of 4 lb.-wt. is then applied at a distance of 1 ft. from the axis, prove that the wheel comes to rest in a little less than 12 sec.

7. A wheel and axle rotates in fixed horizontal bearings without friction and lifts a mass of 100 lb. hanging at the end of a light string wrapped around the axle. At a certain instant the speed of the wheel is 240 revolutions per minute; find the additional height the mass will be lifted from that moment until the wheel comes to rest. The moment of inertia of the wheel and axle about the axis of rotation is 50 lb.-ft.^{2} and the diameter of the portion of the axle on which the string is wound is 6 in.

8. An Atwood's machine carries masses of M at the ends of the string, while the mass of the rider is m. The pulley may be considered to be a single disc, moment of inertia I, radius a. Neglecting friction, show that the acceleration f of the masses is given by

$$\frac{g}{f} = 1 + \frac{2\mathrm{M}}{m} + \frac{\mathrm{I}}{ma^2}.$$

Calculate the acceleration in the case when $\mathrm{M} = 1$ lb., $m = 1$ oz., $a = 6$ in. and $\mathrm{I} = \frac{1}{50}$ lb.-ft. units. What percentage error results from neglecting the inertia of the pulley?

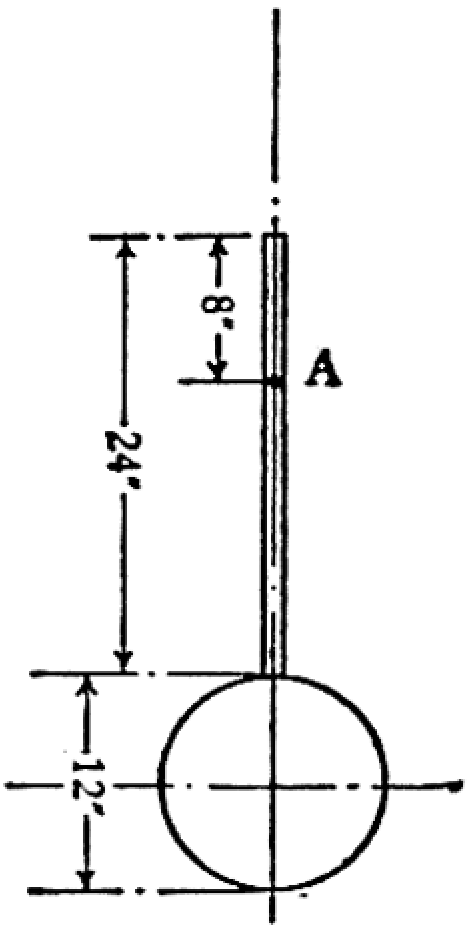
9. A chain, length 20 ft., mass 2 lb. per foot, is hung over a pulley, mass 10 lb., consisting of a uniform circular disc. One end of the chain hangs 2 ft. lower than the other. Find the initial acceleration of the chain, and find its velocity when the lower end has fallen 4 ft.

10. A uniform cylindrical disc, diameter 4 ft., is suspended so that it can swing freely about a horizontal axis perpendicular to the disc and passing through a point O on its circumference. Initially the disc is at rest with its centre C hanging vertically below O. If C is now given a horizontal velocity of 4 ft. per second so that the disc swings as a pendulum about O, find the height to which C will rise above its initial position and the length of the equivalent simple pendulum.

11. A flywheel which consists of a uniform disc, diameter 3 ft., thickness 3 in., is keyed centrally to a circular shaft, diameter 6 in., length 4 ft. The whole rests in frictionless

bearings, the axis of the shaft being horizontal. A light rope wound round the shaft carries a mass M of 200 lb. Initially the system is held at rest. If M is now allowed to descend, thereby causing the flywheel and shaft to rotate, calculate the acceleration of M. Find also the speed of the flywheel in revolutions per minute 10 sec. after the motion commenced. (The flywheel and shaft are both made of steel of density 490 lb. per cubic foot.)

12. The compound pendulum shown in the figure is made up of a thin uniform rod, mass 4 lb., and a uniform flat disc, radius 6 in., mass 1 lb., the whole being suspended from a point A. Dimensions are as shown. Calculate the natural period of the pendulum for a small oscillation in the plane of the disc.



13. A pendulum consists of a heavy uniform circular metal disc, radius 1 in., rigidly fastened at a point of its circumference to one end A of a light rod, length 4 in., the rod being in the direction of the radius produced through

A. The pendulum swings in its own vertical plane about the other end of the rod. Prove that the period of a small oscillation is about 0·724 sec.

14. Three particles, of equal mass, are fixed to a uniform circular hoop at the corners of an equilateral triangle. The hoop being free to swing in a vertical plane about any point in the circumference, show that the equivalent simple pendulum is equal in length to the diameter of the circle.

15. A vertical elastic string connects the lesser mass m of an Atwood's machine to the ground, and the extension of the elastic string when the masses are at rest is a. If the greater mass M is doubled, it is found that the statical extension a is trebled. Prove that $\mathrm{M} = 2m$. If the pulley of the Atwood's machine has a moment of inertia $nmn\,.\,(\text{radius})^2$, show that when the mass $2m$ is pulled down to a point distant $c(< a)$ vertically below its equilibrium position and released from rest, it oscillates over a distance $2c$ keeping time with a simple pendulum of length $(n + 3)a$. (Assume no slip between the pulley and the inextensible string of the Atwood's machine.)

HYDROSTATICS

SECTION T

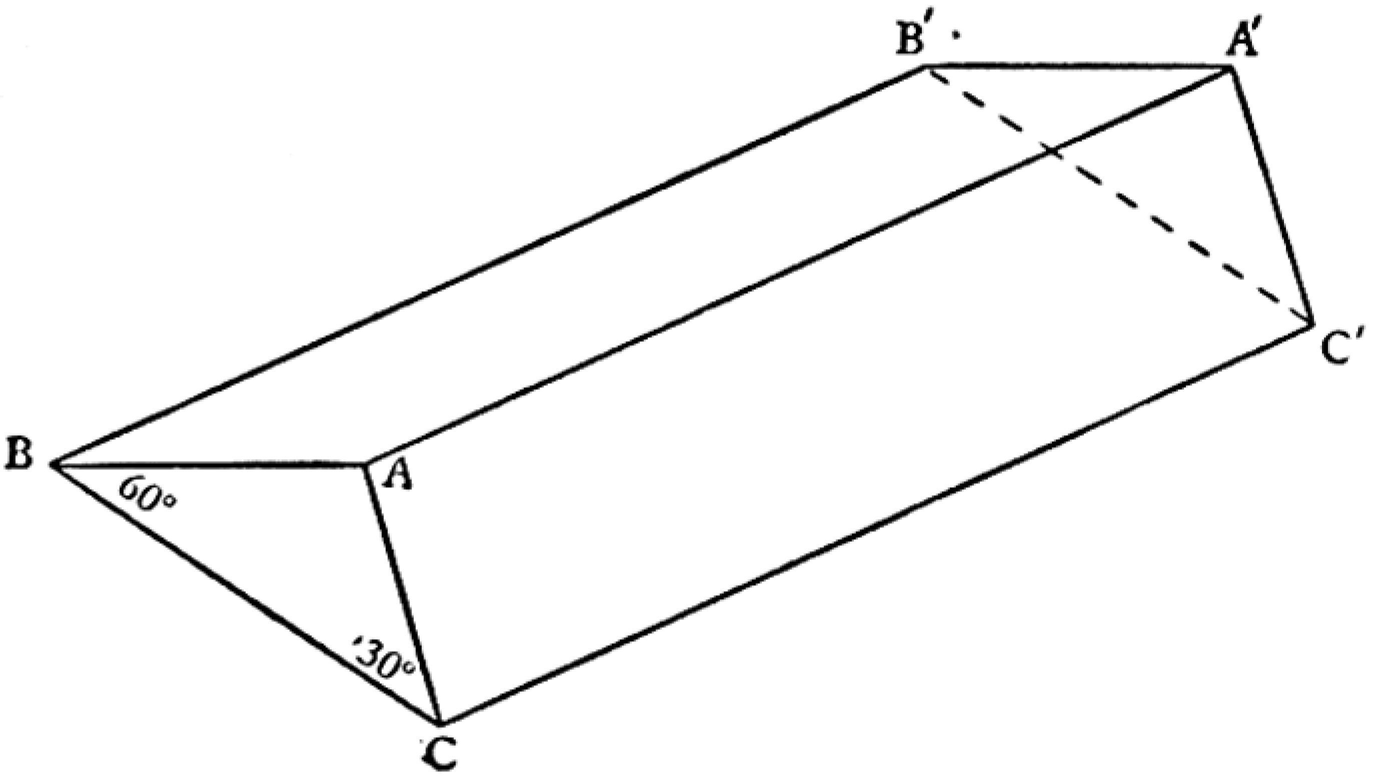
1. A closed cylindrical vessel has base diameter 1 ft., height 3 ft. A tube, diameter 1 in., is fitted into its side and then turns up vertically; water is poured in until it rises to a height of 5 ft. above the base of the cylinder. Find the whole pressure of the water upon the base and on the top of the cylinder. Find also to what height the water would rise in the tube, if the pressure on the top of the cylinder were $\frac{2}{3}$ of the pressure on its base.

2. A cube contains water to a height of 9 in. above its horizontal base, and is filled to the top with oil of specific gravity 0·8. On each vertical face the whole pressure on the part in contact with the oil is equal to the whole pressure on the part in contact with the water. Find the length of the edge of the cube; and deduce the total fluid pressure on the base.

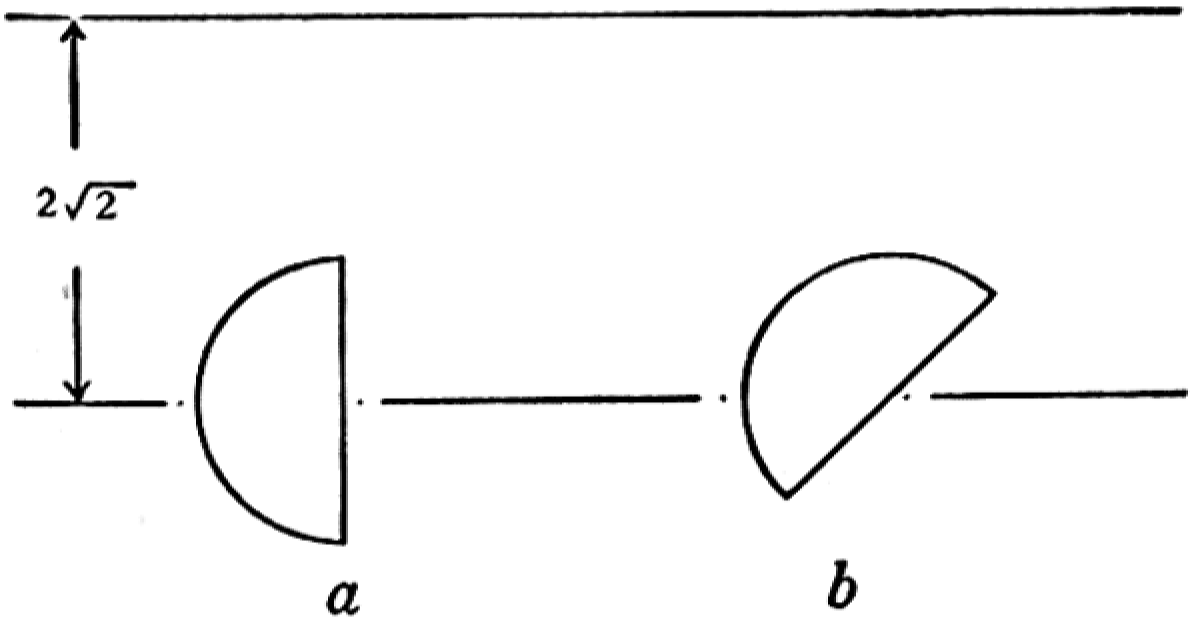
3. A balltap in a cistern consists of a hollow sphere, radius 4 in., mass 8 oz., which floats in the water so that it is just half immersed when the tap is closed. The sphere is attached by a light rod, passing through the centre, to a fixed hinge A on the same level as the centre, the distance from A to the mid-point of the sphere being 18 in. The inlet pipe is closed by a circular valve, radius $\frac{1}{2}$ in., centre 3 in. from A. Find the head of water in the inlet pipe that will just open the valve.

4. A cubical vessel, side 8 in., contains water to a depth of 5 inches. It is turned about one edge of the base until the water is on the point of overflowing. Prove that the resultant pressure on one of the vertical faces is $89{\cdot}6w$, where w is the weight of 1 c. in. of water.

5. ABC is the vertical end-face of a vessel whose base is the horizontal plane through BC. Compare the pressures on ABC in the two cases when the vessel is (*a*) completely, (*b*) half, full of water.



6. The figure shows a solid hemisphere, radius 1 ft., immersed in water so that the centre of its plane face is at a depth $2\sqrt{2}$ ft. In case (*a*) the plane face is vertical; in case (*b*) it is inclined at 45° to the vertical. Find in each case the direction of the resultant liquid thrust on the curved surface of the hemisphere, expressing it in terms of the tangent of the angle made with the horizontal.



7. A vertical trap-door in the form of an isosceles tri-

angle with apex downwards, height 6 ft., base 4 ft., connects two compartments, one containing water, the other containing a liquid of specific gravity 1·6. The level of the water is 2 ft. above the base of the triangle and the level of the liquid is 3 ft. below the base. Find the resultant thrust on the trap-door due to the water and liquid.

8. A right circular cone, vertical angle 60°, is filled with water; it is then closed and placed with a generator in contact with a horizontal table. Show that the resultant thrust upon its curved surface is $\frac{1}{2}W\sqrt{19}$, where W is the weight of water in the cone, and find the direction in which this resultant thrust acts.

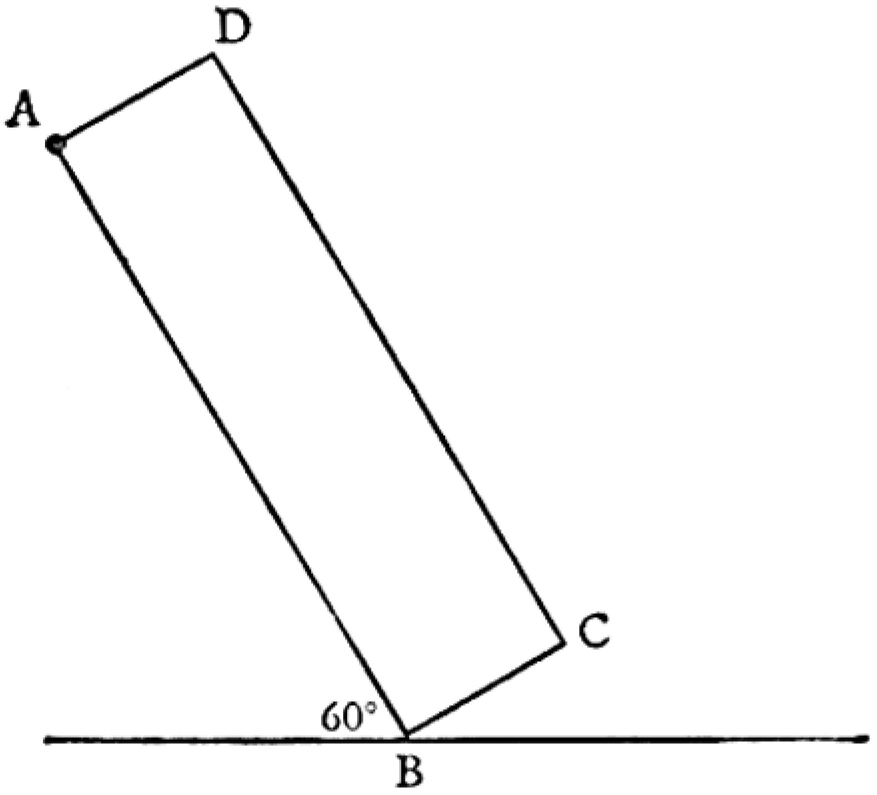
9. Find the depths of the centre of (liquid) pressure in the following cases:

(*a*) A circular lamina, radius x, just immersed, with plane vertical.
(*b*) A trapezium, parallel sides of lengths $2x$, x, vertically immersed with the longer of these sides in the surface and the other at a depth x.
(*c*) A square, side x, immersed with its plane and one diagonal vertical, and a vertex in the surface.
(*d*) The same, partially immersed, the uppermost vertex being at a height $x/2\sqrt{2}$ above the surface.

10. A uniform cube, edge a, floats in water with four edges vertical; the centre of pressure of the immersed part of one face is at a height of $\frac{1}{4}a$ above its immersed horizontal edge. Find the specific gravity of the cube, determining any necessary expression for the depth of the centre of pressure from first principles.

11. Find the depth of the centre of pressure of a rectangular lamina immersed in water so that one edge is in the surface. This rectangular lamina is divided by a horizontal line into two parts such that the total pressures on them are equal. Find the depth of the centre of pressure on the lower of these two parts as a fraction of the depth of the lowest edge of the lamina.

12. The cross-section by a vertical plane of an iron spill-gate, specific gravity 7·5, is a rectangle ABCD and the gate can turn about a horizontal axis through A (see figure). The edge through B is in contact with the base of a canal, and AB makes an angle of 60° with the horizontal. Show that the gate will be on the point of turning about the axis when the water reaches half-way up it, if $\frac{BC}{AB} = \frac{2-\sqrt{3}}{6}$.



13. A horizontal pipe of circular section, internal diameter 8 in., thickness 0·5 in., is full of water. The pipe is closed by a circular door, diameter 9 in., thickness 2 in., the highest point of one face of the door being hinged to the highest point of the pipe. Prove that, provided the door has specific gravity at least 8·7, the water pressure will not open it.

14. Find the position of the centre of pressure of an equilateral triangle immersed in water, in a vertical plane with its base horizontal and vertex uppermost. P, Q, R are the mid-points of the sides DA, AB, BC of a square lamina ABCD, side $2a$. The triangles APQ, BRQ are removed and the remainder is suspended vertically in water, with Q in, and DC parallel to, the liquid surface. Find the ratio of the liquid pressures on the portions

PQR, PRCD, and find the depths of the centre of pressure of each portion.

15. A door in the vertical side of a cistern can open by turning about a horizontal axis through its centre of gravity; it is prevented from doing so by a bolt at a point of its perimeter. Prove that (whatever be the shape of the door) when the door is entirely below the surface of the water, the force on the bolt is independent of the depth of the water. If the door is a square, side 9 in., two of whose sides are horizontal, and the bolt is in the lowest side, show that the force on the bolt is about $4\frac{1}{2}$ lb.-wt.

SECTION U

1. Equal volumes of three liquids, specific gravities 0·80, 1·20, 1·55, are mixed, the whole contracting 8 per cent in volume. Find the specific gravity of the mixture. Find the corresponding value had the mixture been of equal parts by weight, with the same percentage contraction in volume.

2. A body weighs 15·6 gm.-wt. in air, and its apparent weights in water, in a liquid, and in a dilution of the liquid are 13·6 gm.-wt., 13 gm.-wt., and 13·05 gm.-wt. respectively. Determine the specific gravities of the liquid and the dilution, and calculate the percentage volume of the liquid in the dilution assuming that no contraction takes place.

3. A piece of iron (specific gravity 7·8) is fastened to a piece of lead (specific gravity 11·3). Together they weigh 930 gm.-wt. in air, 830 gm.-wt. in water. Find the ratio of the volumes of the pieces of metal.

4. A cube, edge 4 ft., made of cement, specific gravity 2·5, is being hoisted from the bottom of a lake by a cable which will just support a load of 8000 lb.-wt. without breaking. What volume of the cube will be above the surface of the water when the rope breaks?

5. A ship which, with its cargo, weighed 4100 tons-wt.,

sank 3 in. on passing from the sea into a fresh-water canal. Discharge of 50 tons-wt. of the cargo caused it to rise $1\frac{1}{2}$ inches. What value does this give for the specific gravity of sea-water? Assume that the ship's sides are vertical in the water-plane.

6. Two solid uniform spheres, each of radius 4 in., are connected by a light string and totally immersed in a tank of water. If the specific gravities of the spheres are 0·4 and 1·8, find the tension of the string and the pressure between the bottom of the tank and the heavier sphere.

7. A solid sphere, mass M, specific gravity s_1, is fastened beneath the surface of a liquid, specific gravity $s_2 (> s_1)$, by a string attached to the lowest part of the sphere and to the base of the containing vessel. Find the tension of the string. The liquid slowly drains out of the vessel. If $s_2 = 3s_1$, show that, when half the sphere is above the surface, the tension is reduced to one-quarter of the original value.

8. A cylinder of wax is allowed to float in water contained in a graduated jar. The cylinder is then recast so that its length is doubled while the area of cross-section is halved. How is the level of the water in the jar affected? If the height of water in the jar rises from 30 cm. to 34 cm. when the wax floats and to 36 cm. when it is submerged, find the force required to submerge it, its mass being 20 gm.

9. A hollow open metal cone of negligible thickness, having height 2 ft., base radius 1 ft., mass 16·2 lb., floats in water with axis vertical and vertex downwards. Find the depth immersed. If water is slowly poured into the cone, find the depth of liquid it contains when it is about to sink.

10. A balloon, volume 500 c.m., is filled with hydrogen at the same temperature and pressure as the surroundings (viz. N.T.P.). What is the maximum load it will lift at the earth's surface where density of air = 0·00129 gm./c.c. and density of hydrogen = 0·00009 gm./c.c.?

11. A piece of iron (specific gravity 7·8) is embedded in a spherical lump of ice (specific gravity 0·918), radius 10 cm., which floats in water with $\frac{1}{20}$ of its volume above the surface. Find the volume of iron. If the ice slowly melts, its shape remaining spherical, find the radius of the sphere when it sinks.

12. A hollow cylinder made of material of negligible thickness is open at the top and floats in water with its axis vertical and half the cylinder immersed. Oil, specific gravity 0·4, is now poured into the cylinder until the oil surface inside the cylinder is exactly level with the water surface outside. Prove that $\frac{5}{6}$ of the cylinder is immersed.

13. A test-tube is cylindrical with a hemispherical end. Its total length is 11 cm., its diameter is $1\frac{1}{2}$ cm. and its mass 5 gm. When the hemispherical end is just full of mercury (specific gravity 13·6), the tube floats vertically with its open end in the surface of an oil. What is the specific gravity of this oil? What extra weight of mercury must be poured in if the tube is to float similarly immersed in water? (The thickness of the tube may be neglected.)

14. A hollow ball, radius 1 ft., of material having specific gravity 8 and uniform thickness t in., floats in water with its centre on the level of the surface. Find t.

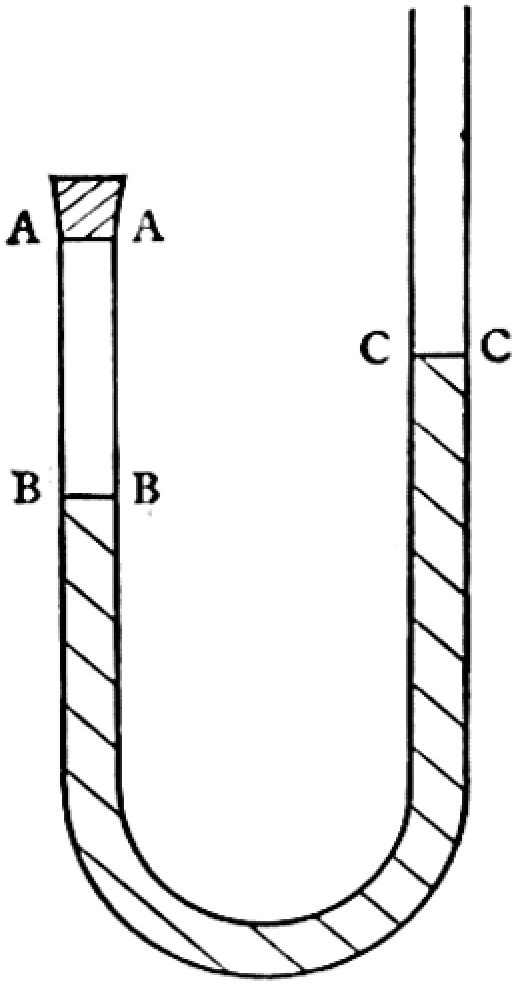
15. A cylinder, mass 10 gm., has a piece of metal (specific gravity 7·5) attached to the centre of one of its ends, the mass of the metal being 1·5 gm. The whole is floated in water with the axis of the cylinder vertical and the metal underneath, the level of the water surface being marked on the cylinder. The metal is now removed and the cylinder floated with its axis vertical in another liquid, it being found that the mark on the cylinder is again in the surface, the cylinder being the same way up. Find the specific gravity of the liquid correct to 3 significant figures.

16. A hollow right circular cylinder, radius 2 ft., height 4 ft., stands with its axis vertical and is half filled with water. A uniform wooden sphere, radius 1 ft., made of material of specific gravity 0·7, is placed in the water.

Find the distance through which the surface of the water rises.

17. An airtight piston, mass M, slides smoothly in a cylinder whose axis is vertical; it is at rest at a height h from the base of the cylinder, being supported by the compressed air below it. An additional mass M is placed on the piston and the new position of equilibrium is at a depth k below the former. Show that the pressure of the compressed air is now $\frac{h}{h-2k}$ times atmospheric pressure.

18. A U-tube of uniform cross-section 1 sq. cm. contains mercury between the levels BB, CC, while the space above BB is filled with air (see figure). The vertical distance AB = 16 cm., and CB = $\frac{1}{2}$AB. When 14$\frac{1}{2}$ c.c. of mercury are poured into the open end of the tube, AB becomes 14·2 cm. Show that the barometric height is very nearly 78 cm.



19. A cylindrical vessel, closed at the upper end and open at the bottom, height 10 ft., cross-section 79·2 sq. ft., is

lowered with its axis vertical to the bed of a lake, its upper end being 28 ft. below the surface of the water. If the height of the water barometer is 33 ft., find the number of cubic feet of air at atmospheric pressure which must be pumped in to expel the water, and the volume of water so expelled.

20. A hollow cylindrical vessel, height h, is lowered vertically into water with its axis vertical and open end downwards, until the open end is at a depth x. Prove that if y is the height to which the water rises within the cylinder, and H is the height of the water barometer,

$$x(h - y) = y(\mathrm{H} + h - y).$$

If $\mathrm{H} = 33$ ft., $h = 3{\cdot}5$ ft., (i) obtain the value of x when the pressure within the cylinder is five-fourths of the atmospheric pressure; and (ii) show that when the top of the cylinder is in the surface, the height of the water within the cylinder is approximately 3·7 in.

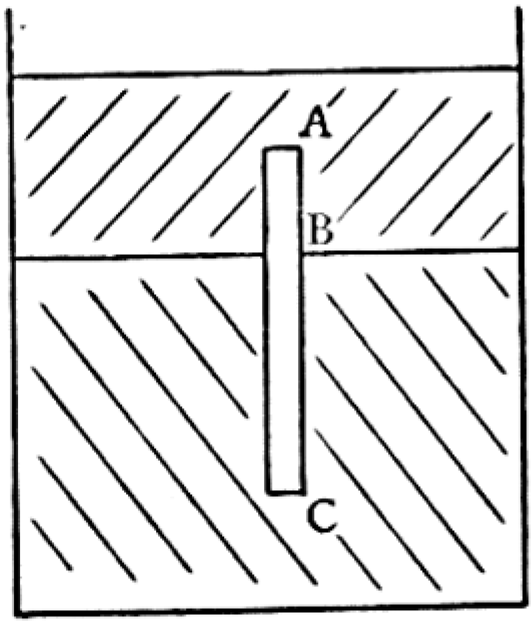
SECTION V

1. A thin cylindrical stick consists of two parts, equal in length. The specific gravity of one part is $\frac{5}{9}$. Show that the least specific gravity of the other part that will allow the stick to float upright in stable equilibrium in water is $\frac{10}{9}$.

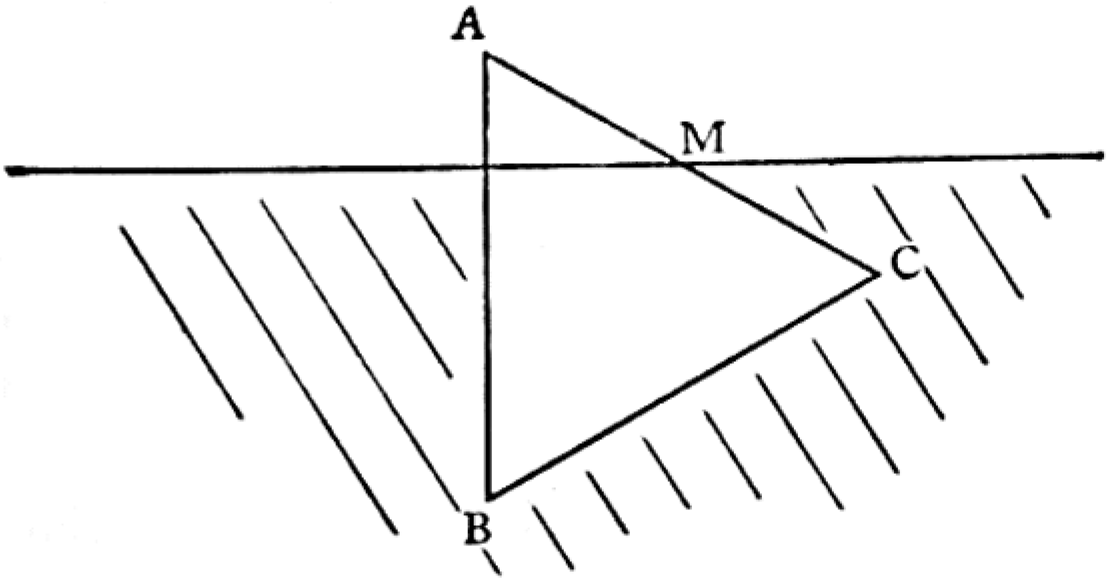
2. A rectangular solid, specific gravity s, height h, base a square of side a, floats freely in a liquid of specific gravity k, the containing vessel being rectangular and having as base a square of side b. A liquid of specific gravity $\sigma(< k)$ which does not mix with the former liquid is poured into the containing vessel until the top of the rectangular solid is in the upper surface of the liquid. Prove that the solid has risen a height

$$\frac{\sigma(k - s)h(b^2 - a^2)}{kb^2(k - \sigma)}.$$

3. The figure shows a rod ABC, of uniform cross-section, floating vertically in equilibrium in two liquids which do not mix, their common surface containing the section of the rod through B. The liquids have specific gravities 0·8 and 1·4; B divides AC in the ratio 2 : 1. Find the specific gravity of the rod, in the two cases when the greater portion of the rod is (i) in the upper, (ii) in the lower, liquid.



4. ABC is an equilateral triangular plate, of material having density 0·75 gm./c.c. With A as point of suspension it hangs freely, dipping into a liquid of density d gm./c.c. (see figure). In the position of equilibrium AB is vertical, and the mid-point M of AC is in the liquid surface. Find the value of d.



5. A particle, mass M, is attached to the corner A of a rectangular disc ABCD, mass N, specific gravity $\frac{1}{4}$. The

disc is fastened by a string, attached at C, to the bottom of a vessel of water in which it floats in a vertical plane with the diagonal BD in the surface. Prove that

$$N = 6M.$$

6. A uniform log, whose cross-section ABCD is a square, can turn freely about its edge through A, which is fixed in the free surface of a liquid of specific gravity σ. The log floats with B above A and ABCD vertical. If AD makes an angle of 30° with the free surface of the liquid, prove that $\rho : \sigma = 7 : 9(\sqrt{3} + 1)$, where ρ is the specific gravity of the material of the log.

7. A rectangle ABCD, in which $AB = \sqrt{2} . BC$, represents a vertical face of a rectangular solid, specific gravity s, floating in stable equilibrium, AB being under water; E and F are the mid-points of BC and CD. E is fastened to one end of a thread, the other end of which is fastened to a fixed point at the bottom of the water. If the body comes to rest with ADF above the surface of the water, show that $s = \frac{2}{3}$.

8. A wine glass when filled with water holds one-fifth of a pint; its rim is a circle of diameter $2\frac{1}{4}$ in.; the greatest depth of the contained liquid is $2\frac{2}{3}$ in. The glass, so filled, being inverted on a horizontal plane, prove that equilibrium is impossible if the mass of the empty glass is less than about 2·13 oz.

9. A cylinder, base radius 3 in., is made of uniform material, specific gravity 0·25. Show that the cylinder will float in stable equilibrium with its axis vertical, when its height is less than 4·9 in.

10. In a ship of 10,000 tons displacement the moving of 20 tons from amidships a distance of 40 ft. across the deck causes the bob of a pendulum 20 ft. long to move through 1 ft. Prove that the metacentric height is 1·6 ft.

11. A rectangular vessel, whose base is a square of side a and whose height is h, is open at the top and made of thin material whose weight and volume may be neglected.

It stands in a rectangular container whose base is a square of side b. Water is poured into both vessels until the inner one is just filled and just covered. Prove that the work done in raising it so that its base is just clear of the water is

$$\frac{wh^2a^2(b^2 - a^2)}{2b^2},$$

where w is the weight of unit volume of water.

12. A ball, radius a, weight W, specific gravity 1·5, lies at the bottom of a cylindrical tank, base radius $2a/\sqrt{3}$. Water is poured in to a depth $d(>2a)$. Find the work done in raising the ball just clear of the water.

ANSWERS

STATICS

SECTION A

1. $242\frac{1}{2}$ gm.-wt. 2. (*a*) $3\frac{25}{46}$ ft., $1\frac{4}{13}$ ft., (*b*) $3\frac{65}{86}$ ft., $1\frac{4}{13}$ ft. 3. 24, 4 lb.-wt. 6, 1 lb.-wt. 5. $36\frac{2}{3}$, $63\frac{1}{3}$ cm. 6. $\frac{59}{6 - x}$ lb.-wt. at C, $\frac{49 - 18x}{6 - x}$ lb.-wt. at B; $1\frac{1}{12}$ ft. 7. $3\frac{1}{3}$ ft. from A. 8. (i) AC = 15 cm., (ii) AC = 30 cm. 9. $\frac{1}{2}$ in. or 17 in. from the 3-lb. mass. 10. $72\frac{1}{2}$ lb.-wt. 11. $4\frac{4}{5}$ ft. 12. 1 ft. 13. $\frac{W}{2c}\{2nx + (1 - 2n)b + c\}$ and

$$\frac{W}{2c}\{(2n + 1)c + (2n - 1)b - 2nx\};\ (4n + 1)b.$$

14. $\frac{67}{56}$ W, $\frac{45}{56}$ W. 15. $\frac{1}{4}$(W + 2P), up or down.

SECTION B

1. 42·8 lb.-wt. per square inch. 2. 2·8. 3. $2\frac{7}{30}$ in. from the end. 4. $111\frac{1}{9}$ lb.-wt.; 108 lb.-wt. 5. $\frac{7}{20}$. 6. $\frac{13\pi}{14} = 2{\cdot}92$ in. 7. $\frac{25}{9\pi} = 0{\cdot}884$ lb.-wt. 8. 35,700 lb.-wt. 9. $15\frac{5}{21}$ lb.-wt. 10. $153\frac{9}{143}$. 12. $(M - m_3) : (m_4 - M)$. 13. 20 lb.; 7 lb. 14. $2\frac{7}{11}$ in. and $7\frac{4}{11}$ in. from left and bottom edges. 16. 3·54 ft. 17. $4\frac{2}{9}$ in., $5\frac{5}{9}$ in. 18. $1 : \sqrt{6}$.
19. $r\sqrt{3}$. 20. 11·6 cm. 21. 2·93 in., 3·34 in.
22. 4·95 in. 23. (6·03, 6·03). 24. $\frac{7}{45}$ AD.

SECTION C

1. 6 lb.-wt. along OE. 2. 0·766, 0·401. 3. $\frac{1}{2}$ lb.-wt., $\sqrt{3}/2$ lb.-wt. 4. $2\sqrt{2}$W; $\sqrt{5}$W at $\tan^{-1}\frac{1}{2}$ above the horizontal.

5. 76° 31′ or 103° 29′. 6. 4·8 in. 7. At $\cos^{-1}\frac{2}{3}$ with the horizontal; 149 lb.-wt.; $\sin^{-1}\frac{1}{9}$. 8. $5y/3x$. 9. W.

10. $W\sqrt{7}$; $\dfrac{2\sqrt{3}a}{21}$. 11. $\dfrac{(25 + 3\sqrt{3})a}{32} = 0{\cdot}94a$. 13. 33·6 ft.

14. 4·2 lb.-wt. 15. r. 16. $\tan^{-1}3\sqrt{3}$; $\dfrac{3W\sqrt{7}}{14}$.

18. $\tan^{-1}7$. 20. $\dfrac{\sqrt{37}\,.\,W}{6}$ and $\dfrac{W}{6}$; $\dfrac{W}{6}$. 21. $\dfrac{2\sin\alpha}{\sqrt{5 - 4\cos\alpha}}\,W$.

22. 30° 43′. 24. 6 in. 25. 15 in., $1\frac{7}{8}$ lb.-wt., $1\frac{5}{8}$ lb.-wt.

SECTION D

1. 26° 34′. 2. (a) 14·1 lb.-wt., (b) 3·2 lb.-wt.

3. $\pm\,0{\cdot}0208$ lb.-wt. 4. $\dfrac{89W}{120}$. 5. $\dfrac{W\sin\alpha}{2\sin(\alpha + \beta)}$; $\dfrac{W\sin\alpha}{2}$.

6. 30°, $1/\sqrt{3}$. 7. $\dfrac{W\tan\theta}{4}$ and $\dfrac{3W}{4}$. 10. $2a/5$. 12. 7·37 lb.-wt.; at 17° 27′ with P. 14. $\dfrac{a}{2 - \mu}$; $\dfrac{a}{4 + \mu}$. 15. 39°.

SECTION E

1. 2 lb.-wt., $2\frac{1}{2}$ ft.; 1 lb.-wt., 14 lb.-ft. 2. 72 lb.-ft. 3. $5\frac{1}{3}$ lb.-ft. clockwise. 4. $4\sqrt{3}$ lb.-ft; 3 ft. from B.

5. 6P at 60° to AB; $\dfrac{21\sqrt{3}\,aP}{2}$. 6. 13 lb.-wt.; $1\frac{3}{26}$ ft.

7. $\sqrt{5}P$; $6a$ from A. 8. $2\sqrt{5}$ lb.-wt.; 9 ft. from A on BA produced, and 18 ft. from A on AD produced. 11. 6 lb.-wt.; $\dfrac{8\sqrt{3}}{3}$ in. 12. 3. 13. 10P; $3x + 4y = 18a$. 14. $\frac{1}{5}$ ft., $\frac{1}{14}$ ft.; $5x - 14y = 1$. 15. 30·4 lb.-in. 16. 28 lb.-ft. 17. $5\sqrt{2}$ lb.-ft.

SECTION F

1. 1 in $11\frac{1}{19}$. 2. 1 in.; $\frac{4}{5}$, $\frac{11}{60}$, $\frac{1}{60}$ ton-wt. 3. $W = \dfrac{2cP}{a - b}$; 40 lb.-wt. 4. 280; 20 lb.-wt. 5. 5; 2·92, 58·3 per cent.

6. 3·19 ft.; 0·042 ton-wt. 8. $\frac{12}{7}$. 9. 4·78 cm., 2·84 cm. 10. 3 in., $4\frac{1}{2}$ in. 11. $\sin^{-1}\frac{1}{3}$. 12. 3·77 in. 13. $r/2$, $5r/22$ from centre of solid. 15. $4r/9\pi$ from centre, in heavier part. 16. 21·3 in. 17. $\tan^{-1}\dfrac{28}{15\pi} = 30° 43'$. 19. 5·5 in., 10·4 in.

SECTION G

1. $\dfrac{3\sqrt{3}W}{2}$ horizontally; $\dfrac{W}{2}$ vertically. 4. $1\frac{2}{5}$ lb.-wt. 6. $33\frac{1}{3}$ lb.-wt.; 0·857. 11. $\tan^{-1}\frac{1}{6}$. 12. $\dfrac{a(\sin\alpha - \mu\cos\alpha)}{1 + \sin\alpha - \mu\cos\alpha}$. 16. $\alpha + 2\lambda$. 17. 0·654. 19. Leading block in direction of rotation; 15° 58′. 21. Tensions in AB, BC, CD, DE respectively $8\sqrt{2}$, 8, 6, $6\sqrt{3}$ lb.-wt. F = 4·1 lb.-wt. at 46° 56′ below horizontal. 25. 1599 lb., 615 lb.-wt. 28. $n = \sqrt{3} + 1$.

SECTION H

1. 3 ft.; 5W/6. 2. Compressions: AB 5·8 cwt., AC 11·6 cwt. Tensions: BC 11·6 cwt., CD 11·6 cwt. 3. Reactions: W/2 horizontally. Tension, 2W. 4. Compressions: AB 75 lb.-wt., AD 125 lb.-wt., BC 125 lb.-wt. Tension: CD 75 lb.-wt. 5. 2700 lb.-wt.; 3550 lb.-wt. at $58\frac{1}{2}°$ with horizontal.

6. 165 lb.-wt. at 21° above horizontal. 7. Compressions: BC 40 lb.-wt., CD 46·5 lb.-wt., AC 23 lb.-wt. Tension: AB 35 lb.-wt. Reaction at A, 31 lb.-wt. downwards at 40° to horizontal. Reaction at D, 46·5 lb.-wt. upwards in line of DC. 8. 35 cwt., 45 cwt. Compressions: AB 49·5 cwt., BC 40 cwt., CD 63·6 cwt., CE 7·1 cwt. Tensions: AE 35 cwt., ED 45 cwt., BE 7·1 cwt. 9. Compressions: AB 1 ton-wt., BC 1 ton-wt., AD 1·4 tons-wt., CE 1·4 tons-wt., BD 4 tons-wt., BE 4 tons-wt. Tension: DE 4·4 tons-wt. 10. Compressions: AC 34·6 cwt., CD 20 cwt., BD 60 cwt., DE 40 cwt. Tensions: AE 17·3 cwt., CE 20 cwt., EF 52 cwt., FB 52 cwt. No stress in DF. 11. Reaction: 18·5 cwt. at 39° to horizontal, towards D. Compression: BD 23 cwt. Tensions: AB 16·5 cwt., CD 5·9 cwt., BC 11·5 cwt. 12. Compressions: AD 12 cwt., CD 16 cwt.

Tensions: AB 9·6 cwt., BD 8 cwt., BC 9·6 cwt. 13. Compressions: AB 3460 gm.-wt., BC 1730 gm.-wt., BD 3000 gm.-wt. Tensions: AC 4000 gm.-wt., CD 3460 gm.-wt.
14. Reaction: 1·85W at 33° to horizontal, T = 1·5W.
Compressions: OA 0·57W, OB 0·67W, OC 0·77W.
Tensions: AB 1·15W, BC 1·33W. 15. Compressions: AB 7 tons-wt., BC 7 tons-wt., BD 8 tons-wt. Tensions: AC 9·8 tons-wt., CD 7 tons-wt. 16. Reaction at A, 1·6W downwards to the right from 19° above horizontal. Reaction at B, 2·15W upwards to the left from 45° below horizontal.
Compressions: FC 1·4W, CE 0·7W, AB 1·5W, BC 1·5W, CD 0·5W. Tensions: AD 0·5W, DE 0·7W, EF 1·0W.
17. P = 5·15 lb.-wt. Compressions: EF 1·6 lb.-wt., FD 1·6 lb.-wt. Tension: DE 0·9 lb.-wt.
18. Force: $35\frac{1}{2}$ lb.-wt. Reaction: 29 lb.-wt. upwards to the right at 30° with horizontal. Compressions: AF 17 lb.-wt., FC 28·5 lb.-wt., CE 28·5 lb.-wt., EB 40·5 lb.-wt., DF 12·5 lb.-wt., FE 11·5 lb.-wt. Tensions: BD 20 lb.-wt., DA 33 lb.-wt., ED 12·5 lb.-wt.
19. Inclinations of AB, BC: 53° 8′, 33° 41′. Tensions: AB 2·5W, BC 1·8W, DE 1·8W, EF 2·5W. Length of BC = 3·6 in.
20. Tensions and inclinations to horizontal of portions under AB, BC, CD, DE respectively, 17·2 lb.-wt., 30°; 15·2 lb.-wt., 13·5°; 15·4 lb.-wt., 13°; 17·6 lb.-wt., 32·5°.

SECTION J

2. Compressions = $T/\sqrt{3}$. 3. $13w$. 4. 0·007 cm.
5. 9·91 cm. 8. 17·9 lb.-wt. 9. 13·1 in. 10. $\frac{1}{20}$ in., nearly. 11. 0·031 cm. 13. $\frac{15Wa}{16}$. 14. $\frac{3Wa}{4}$.
15. 2·4 ft. 16. $3\frac{3}{4}$ ft. from A.

DYNAMICS

SECTION K

1. 16 min.; $6\frac{5}{8}$ miles. 2. 10 ft. per second per second;
7 sec.; 70 ft. per second. 3. $45\frac{5}{11}$ miles per hour $=$ $66\frac{2}{3}$ ft.
per second. 6. (*a*) 2 hr. $2\frac{1}{2}$ min.; (*b*) 1 hr. $43\frac{1}{2}$ min.
7. 9735 ft. 8. 30 and 33 ft. per second. 9. 0·65 ft. per
second per second. 11. 1089 ft. 12. 48 sec. 13. 6300 ft.
15. 2900 ft.; 0·43 ft. per second per second. 17. 3·2 sec.
after passing 20-ft. mark; or approximately 10·7 sec. from the
commencement. 18. 16 sec.

SECTION L

1. $1{\cdot}69 \times 10^{-10}$ dynes. 2. 65·4 sec. 3. 20·2 lb.-wt. in
a direction bisecting the right angle. 4. 20·7 ft. per second.
6. $170\frac{1}{2}$ ft. from the station. 7. A stops 0·6 ft. from C.
8. 1700 ft. 10. (i) 11·3 ft.; (ii) 45·9 ft.
12. (*a*) $g/17$; (*b*) $g/23$. 13. 3·95 sec.
14. $4\frac{4}{5}$ lb.; 19·2 ft. per second per second.

SECTION M

1. (*a*) 273 miles per hour; (*b*) 5° 40′ north of west.
2. 7·21 miles per hour from 11° 19′ south of west.
3. $180\sqrt{3}$, 30° north of east; $60\sqrt{21}$, 49° 6′ north of west;
$60\sqrt{39}$, 13° 54′ west of south. 4. 2π ft. per second.
5. (i) 12·65 miles per hour; (ii) 8 miles per hour; (iii) 15·6
miles per hour. 6. $\frac{8t}{2-k^2}$. 7. At $\tan^{-1}\frac{s}{b}$ with the line of
traffic. 8. At 48° 35′ with the line FW. 10. 19° 28′ north
of east; 1·57 hr. 19° 28′ south of east; 3·28 hr. 11. 75 miles
per hour; 55·4 min., 89·7 min. 12. $2\frac{1}{4}$° west of south;
13.42 hr. 13. 4.47 p.m.; 11·5.

Section N

3. 130 ft. 4. 25 ft.; 100 ft. 5. 100 ft. per second, at 19° 48′ above horizontal. 6. 70° 30′ above and 33° 40′ below the horizontal. 8. 75°. 9. 44·2 ft. per second; 50° 40′. 10. Initial velocity $80\sqrt{2}$ ft. per second at $\tan^{-1}\frac{1}{3}$ above the horizontal. 11. 45°. 12. 7·24 ft. Less. 13. 45° or 66° 48′. 16. 172·6 ft. per second; AD = 720 ft. 19. $h\sqrt{n}\sqrt{n+1}$. 25. 60° to horizontal; 8 : 3. 27. (*a*) $52\frac{1}{12}$ and $104\frac{1}{6}$ ft. (*b*) $50\sqrt{2}$ ft. per second.

Section P

2. 310 foot-pounds. 3. 188 horse-power. 4. 29·8 horse-power. 5. Cost by electricity one-half that by petrol. Accelerations, e.g. from stops. 7. 22,400 foot-pounds; 793 watts. 8. 28 horse-power. 9. 0·1725 horse-power. 10. $\frac{2}{7}$ ft. per second per second. 11. 470 horse-power; $\frac{8}{25}$ ft. per second per second. 12. 672 horse-power; $\frac{5}{288}$ ft. per second per second. 13. 76 lb.-wt.; 59·6 miles per hour. 15. 1 in 38·4. 16. R = 12; 236,544,000 foot-pounds = 20 mile-tons. 17. 11·9, 30·2, 48·4 and 60·3 ft. per second; 1·11 ft. per second per second. 18. 1880 horse-power. 20. (i) 1·25 sec.; (ii) 7·5 ft.; (iii) 22·5 foot-pounds. 21. 50 miles per hour; 0·134 ft. per second per second. 22. 37·5 inch-pounds. 23. 12·5 and 14 ft. per second; 18·4 horse-power. 24. 34·9 horse-power; $4\frac{1}{11}$. 25. $\frac{15}{32}$; $\frac{48}{550}$ horse-power.

Section Q

1. 0·17. 5. No difference. 6. Velocities of 8 and 5 lb. masses, respectively, $\dfrac{30 - 100e}{13}$ and $\dfrac{30 + 160e}{13}$ in the original direction of the larger mass. 7. $\frac{1}{3}$; 5 : 7. 10. 10 ft. per second; 1·25 foot-tons; 0·527 ft. 12. $\frac{1}{2}mg(h - a)$. 13. 187·5 f.p.s. units; 6·87 foot-pounds; 29·3 lb.-wt. 14. 5128 ft. per second. 15. 217·2 lb.-wt.; 0·22 sec.; $\frac{10}{11}$. 16. 70·8 f.p.s. units at 31° 59′ with original direction.

18. 1 ft. 4 in. from the wall. 21. 40 ft. 24. 63·13 lb.-wt. 25. (*a*) 2·4 sec.; (*b*) 458 f.p.s. units.

Section R

4. Upper $a(m+n)/h$. Lower an/h. 5. 72·1. 8. 0·252 ft. 9. $60\sqrt{2}$ and $120\sqrt{2}$ miles per hour. 11. $\dfrac{4\pi d}{V}$; $\sqrt{5d}$. 12. 4·16 ft.; 6 sec. 13. $3\sqrt{2}$ in. per second; 2π sec. 15. 1·01 sec.; 31 in. per second. 16. $5\frac{1}{3}$ ft. 17. 1·37 in.; 20 lb.-wt. 18. 0·618 lb.-wt.; 8·66 in. from A. 20. $P\sqrt{a/mT}$. 21. 9 ft. 6 in. 22. 316 ft. per second per second; 10·9 and 12·6 ft. per second. 25. 12·4 in. from C.

Section S

1. 5 sec. 2. 4·71 ft. 4. 13·4. 5. 9·16 sec.; 11·5 revolutions. 6. $600/\pi^2 = 60{\cdot}8$ lb.-ft. 7. 5·55 ft. 8. 0·934 ft. per second per second; 3·88 per cent. 9. 2·84 ft. per second per second; 8·26 ft. per second. 10. $4\frac{1}{2}$ in.; 3 ft. 11. 0·4 ft. per second per second; 153 revolutions per minute 12. 1·43 sec.

HYDROSTATICS

Section T

1. 245 lb.-wt.; 98 lb.-wt.; 9 ft. 2. 31·5 in.; 969 lb.-wt. 3. 76·6 ft. 5. $\dfrac{4(8+5\sqrt{2})}{7} = \dfrac{8{\cdot}6}{1}$. 6. (*a*) $\dfrac{1}{3\sqrt{2}}$; (*b*) $\frac{2}{3}$. 7. 2700 lb.-wt. 8. At 53° 23′ with horizontal. 9. (*a*) $\dfrac{5x}{4}$; (*b*) $\dfrac{5x}{8}$; (*c*) $\dfrac{7\sqrt{2}x}{12}$; (*d*) $\dfrac{79\sqrt{2}x}{200}$. 10. $\frac{3}{4}$. 11. 0·862. 14. 2 : 9; $\dfrac{3a}{4}$; $\dfrac{14a}{9}$.

Section U

1. 1·29; 1·19. 2. 1·3; 1·275; $91\frac{2}{3}$. 3. Iron : lead = 4 : 3. 4. $\frac{1}{2}$. 5. 1·025. 6. 5·82 lb.-wt.; 1·94 lb.-wt.

7. $\frac{M(s_2 - s_1)}{s_1}$. 8. 10 gm.-wt. 9. 11·96 in.; 22·96 in. 10. 600 kgm.-wt. 11. 19·5 c.c.; 7·31 cm. 13. 0·896; 1·98 gm. 14. 0·26 in. 15. 0·885. 16. 2·8 in. 19. 912 c. ft.; 396 c. ft. 20. (i) 8·95 ft.

SECTION V

3. 1·2; 1·0. 4. 0·8. 12. $\frac{Wd}{3}$.